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NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

SENSITIVITY AND ROBUSTNESS ANALYSIS
FOR SEA-SKIMMING MISSILE

by

O avo Amorim de Andrade

September 1986

Thesis Advisor: D. J. Collins

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SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

1a REPORT SECURITY CLASSIFICATION UNCLASSIFIED	1b. RESTRICTIVE MARKINGS
2a SECURITY CLASSIFICATION AUTHORITY 2b. DECLASSIFICATION / DOWNGRADING SCHEDULE	3 DISTRIBUTION / AVAILABILITY OF REPORT Approved for public release; distribution is unlimited.
4 PERFORMING ORGANIZATION REPORT NUMBER(S)	5 MONITORING ORGANIZATION REPORT NUMBER(S)
6a. NAME OF PERFORMING ORGANIZATION Naval Postgraduate School 6b OFFICE SYMBOL (If applicable) 67	7a. NAME OF MONITORING ORGANIZATION Naval Postgraduate School
6c. ADDRESS (City, State, and ZIP Code) Monterey, California 93943-5100	7b. ADDRESS (City, State, and ZIP Code) Monterey, California 93943-5100
8a. NAME OF FUNDING / SPONSORING ORGANIZATION 8b. OFFICE SYMBOL (If applicable)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER
8c. ADDRESS (City, State, and ZIP Code)	10 SOURCE OF FUNDING NUMBERS PROGRAM ELEMENT NO / PROJECT NO / TASK NO / WORK UNIT ACCESSION NO

11 TITLE (Include Security Classification)

SENSITIVITY AND ROBUSTNESS ANALYSIS FOR SEA-SKIMMING MISSILE

12 PERSONAL AUTHOR(S) Andrade, Olavo A.

13a TYPE OF REPORT	13b TIME COVERED	14 DATE OF REPORT (Year, Month, Day)	15 PAGE COUNT
Engineer's Thesis	FROM ______ TO ______	1986 September	120

16 SUPPLEMENTARY NOTATION

17 COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	Sensitivity, Eigenvalues, Robustness, Aerodynamic Coefficients, Sea-Skimming Missile, Optimization, Minimum Singular Value

19 ABSTRACT (Continue on reverse if necessary and identify by block number)

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The study is divided in four basic parts:

a) The development of the model;
b) EIGENVALUE SENSITIVITY ANALYSIS with respect to the variation of the aerodynamic parameters of the autopilot/airframe of the missile;
c) Analysis of the time response with respect to the variation of the aerodynamic parameters; and
d) Robustness analysis and improvement of the system, using the SINGULAR VALUE ANALYSIS.

20 DISTRIBUTION / AVAILABILITY OF ABSTRACT ☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT. ☐ DTIC USERS	21 ABSTRACT SECURITY CLASSIFICATION
22a NAME OF RESPONSIBLE INDIVIDUAL Daniel J. Collins	22b TELEPHONE (Include Area Code) (408) 646-2826 22c OFFICE SYMBOL 67Co

DD FORM 1473, 84 MAR — 83 APR edition may be used until exhausted. All other editions are obsolete

SECURITY CLASSIFICATION OF THIS PAGE

UNCLASSIFIED

19. ABSTRACT Continued

All the analysis is based in results of simulation programs using the software available at the NAVAL POSTGRADUATE SCHOOL.

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Sensitivity and Robustness Analysis for Sea-Skimming Missile

by

Olavo Amorim de Andrade
Lieutenant-Commander, Brazilian Navy
B.S., Escola Naval, R.J., Brasil, 1970
B.S., Universidade de São Paulo, S.P., Brasil, 1976

Submitted in partial fulfillment of the requirements for the degree of

AERONAUTICAL ENGINEER

from the

NAVAL POSTGRADUATE SCHOOL
September 1986

Author: Olavo Amorim de Andrade
Olavo Amorim de Andrade

Approved by: Daniel J. Collins
Daniel J. Collins, Thesis Advisor

Harold A. Titus
Harold A. Titus, Second Reader

Max F. Platzer
Max F. Platzer, Chairman,
Department of Aeronautics

J. Dyer
J. Dyer, Dean of Science and Engineering

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c) Analysis of the time response with respect to the variation of the aerodynamic parameters.

d) Robustness analysis and improvement of the system, using the SINGULAR VALUE ANALYSIS.

All the analysis is based in results of simulation programs using the software available at the NAVAL POSTGRADUATE SCHOOL.

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ACKNOWLEDGEMENT

I wish to express my gratitude to the Brazilian Navy for the opportunity to further expand my knowledge by pursuing this degree.

I would like to express my appreciation to Professor D.J.Collins for his assistance and guidance during this work.

Special thanks to my wife, Ida, and to my children, Renata and Rafael for their encouragement and support that have given me conditions to complete this thesis.

I. INTRODUCTION

The majority of tactical missiles used against surface targets are SEA-SKIMMING and fly as close as possible to the surface of the water in order to make difficult the detection and reaction of the enemy.

This work addresses the problem of analysing the design of the altitude control system of a supersonic SEA-SKIMMING missile in both aspects of SENSITIVITY and ROBUSTNESS, with respect to the variations of the aerodynamics parameters.

Those variations can develop from uncertainties in the model as well as from changes in the flight conditions of the vehicle.

The SENSITIVITY analysis makes it possible to verify, for a given model which parameters are more important for the desired flight path.

The ROBUSTNESS analysis has the objective of verifying if the system could be affected by perturbations or noise and will be the base on calculations of modified feedback gains which are capable of producing the desired flight of the missile despite perturbations.

The contents of the different sections are as following:

Section II presents the model and the state equations of the system as well as the calculations to complete the chain of control of the missile, considering the restrictions of the SEA-SKIMMING scenario, where the altitude control system cannot tolerate any overshoot to avoid that the missile hits the water.

Section III develops the SENSITIVITY ANALYSIS taking into consideration the variation of the eigenvalues and time response of the system of state equations for variations of the aerodynamic parameters in a range of $\pm 25\%$ of the original value.

In section IV is described the ROBUSTNESS ANALYSIS based upon the minimum singular value of the return difference matrix as function of the frequency; section V introduces a design technique using optimization routine in order to obtain a "complete robust" solution and section VI presents the final conclusions.

II. MATHEMATICAL MODEL OF THE MISSILE

The work assumes a SEA-SKIMMING missile with the flight in the vertical plane according to the path shown in Figure II.1.

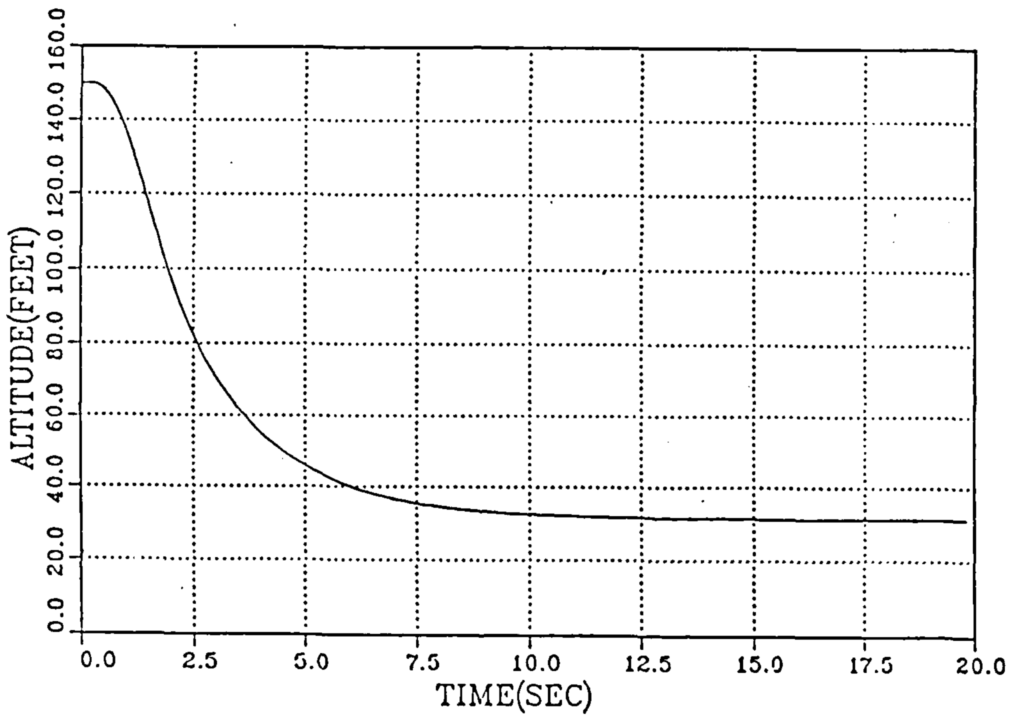

Figure II.1 - Missile Trajectory in the Vertical plane

The missile is assumed to fly at constant velocity, at 150 ft, when it is commanded to a final altitude of 30 ft. Only the pitch channel will be analysed.

A. THE MISSILE

The missile will be considered as presented by Arrow [Ref.1]. This model has been selected in order to avoid classification problems. The geometry as well as its characteristics are presented in Appendix A.

The states that were considered in our model are defined in Table I as well as the relevant aerodynamics parameters.

In the Figure II.2 we have the schematic representation of the uncoupled pitch control chain of the missile, neglecting the yaw and roll movements;

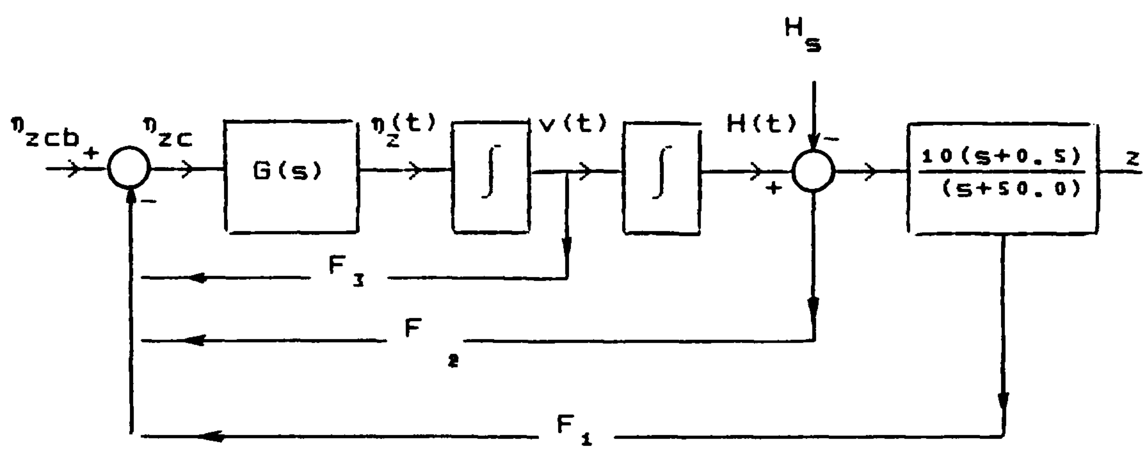

Figure II.2 - Block Diagram of the Pitch Control Chain of the Missile.

where

η_{zcb} - "bias" acceleration;

η_{zc} - commanded acceleration;

η_z - achieved acceleration;

v(t) - velocity;

H(t) - altitude;

H_s - commanded altitude.

The Transfer Function G(s) represents the missile itself and the other blocks are the controller. The input to this block will be the "commanded" acceleration and the output will be the achieved acceleration.

The commanded acceleration comes from the feedback of the states of the controller (compensated state, velocity and altitude) added to a "bias" acceleration.

In the diagram of Figure II.3, we have a complete representation of the system with the states used in modelling our missile.

The "STATE EQUATIONS" which represent our system are the following:

$$
\begin{aligned}
\dot{x}_1 &= -150.0\, x_1 - 2{,}646.66\, x_5 - 705.75\, x_6 \\
\dot{x}_2 &= 1.353\, x_1 + x_3 - 1.353\, \eta_{zc} + 1.334\, H_s \\
\dot{x}_3 &= -6.572\, x_1 - 5\, x_3 + 6.572\, \eta_{zc} - 6.481\, H_s \\
\dot{x}_4 &= -44.332\, x_5 - 59.109\, x_6 \\
\dot{x}_5 &= x_4 - 0.1482\, x_5 - 0.0395\, x_6 \\
\dot{x}_6 &= -188.4\, x_7 + 188.4\, x_8
\end{aligned}
\qquad \text{(II.1)}
$$

$$\mathring{x}_7 = -0.4608\, x_1 - 2.231\, x_2 - 0.3406\, x_3 + 2.231\, x_4 +$$

$$-15.0949\, x_5 - 20.13\, x_6 - 0.143\, x_7 + 0.4608\, \eta_{zc}$$

$$- 0.4544\, H_s$$

$$\mathring{x}_8 = -50.0\, x_8 - 495.0\, x_9 + 495.0\, H_s$$

$$\mathring{x}_9 = x_{10}$$

$$\mathring{x}_{10} = -17.644\, x_5 - 4.705\, x_6$$

This system of equations can be represented in matrix form, where we have:

$$x = A\,\mathring{x} + B\,u$$

$$A = \begin{bmatrix} -150 & 0 & 0 & 0 & 2{,}646.66 & -705.74 & 0 & 0 & 0 & 0 \\ 1.35 & 0 & 1.0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -6.5 & 0 & -5.0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -44.33 & -59.11 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.0 & -0.15 & -0.04 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -188.4 & 188.4 & 0 & 0 & 0 \\ -0.46 & -2.2 & -3.4 & 2.2 & -15.1 & -20.13 & -0.14 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -50. & -495 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -17.64 & -4.71 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ -1.353 & 1.334 \\ 6.572 & -6.481 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0.461 & -0.454 \\ 0 & 495.0 \\ 0 & 0 \end{bmatrix}$$

$$u = -F\,x$$

$$F = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.049 & 0.98615 & 1.404 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The vector input "u" has two components

η_{zc} - the commanded acceleration; and

H_s - the desired altitude.

The input η_{zc} is considered as two parts; one corresponds to the feedback of the states (from the controller) and another is the "bias" acceleration of 1g.

TABLE I
DESCRIPTION OF THE SYSTEM STATES AND
AERODYNAMIC COEFFICIENTS

x_1	filtered acceleration
x_2	acceleration compensator
x_3	acceleration compensator
x_4	q - pitch angular rate
x_5	α - angle of attack
x_6	δ_p - pitch tail incidence
x_7	δ_{pc} - commanded pitch tail incidence
x_8	x_c - controller compensator
x_9	H(t) - altitude
x_{10}	v(t) - velocity

AERODYNAMIC COEFFICIENTS

$C_{m\alpha}$ slope of curve of pitching moment coefficient

$C_{m\delta p}$ change in C_m per degree pitch control incidence

$C_{N\alpha}$ slope of curve of normal force coefficient $C_N \times \alpha$

C_{Np} change in C_N per degree pitch control incidence

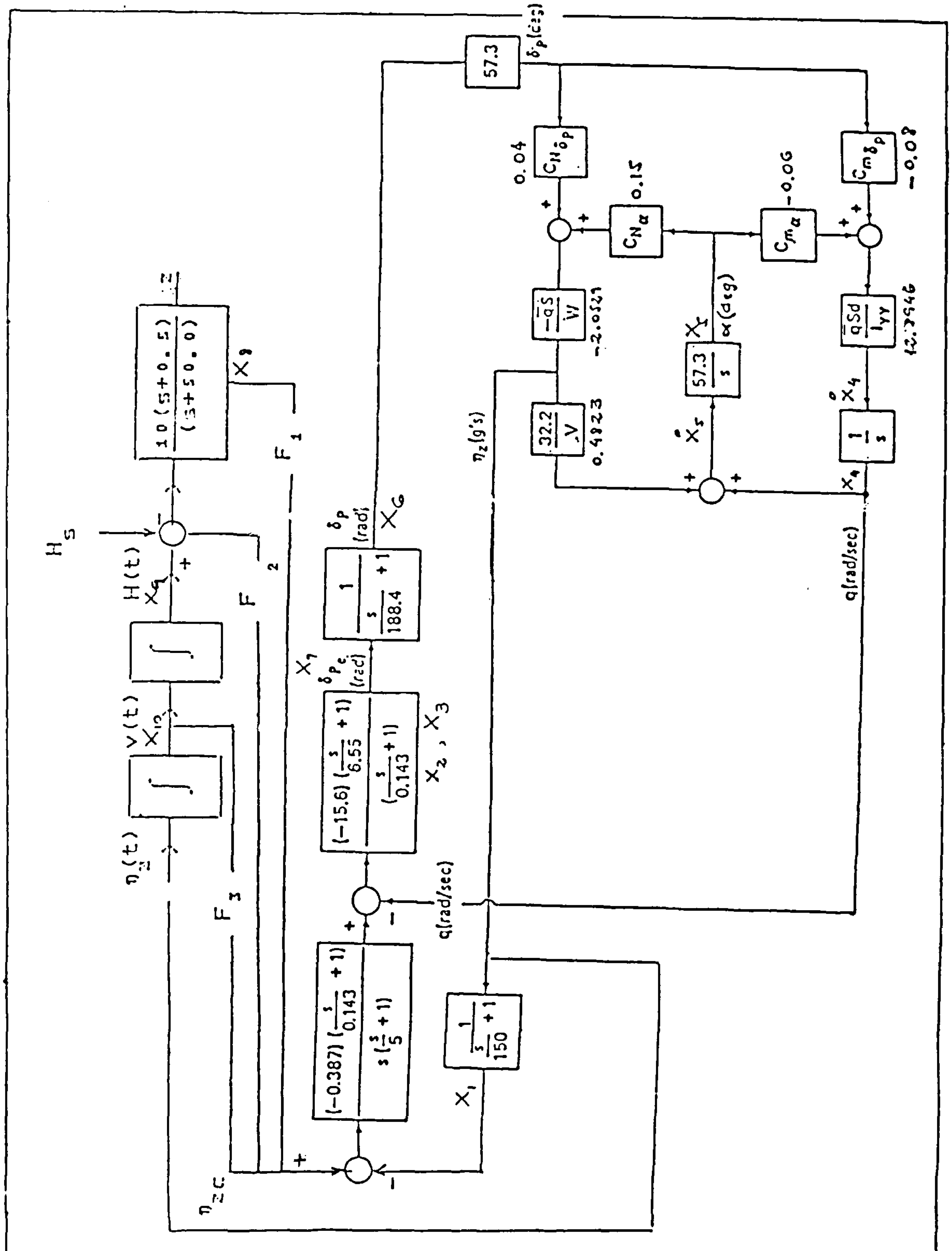

Figure II.3 - The Missile Model

The model can be separated in two main parts; the CONTROLLER and the AUTOPILOT/AIRFRAME.

B. THE CONTROLLER

The controller was developed based in the procedure introduced by Dowdle [Ref.2].

The missile was considered as a point mass and, therefore the altitude as function of time is calculated by double integration of the achieved acceleration.

The controller was developed without take in consideration the missile itself, i.e., the transfer function G(s) in Figure II.2 was assumed as "unity".

According to the [Ref.2], this procedure is valid if the bandwidth of G(S) is larger than the crossover frequency of the controller.

Due to the constraints of the scenario, the controller has to be designed in such way that no overshoot will be tolerate. Further requirements are that the angle-of-attack and the acceleration, have to remain below a certain level.

The controller are responsible for the commanded acceleration by feedback of its states.

The technique used to solve the feedback problem was the LINEAR QUADRATIC STATE FEEDBACK REGULATOR. This solution has a large stability margin with respect to phase variations, as well infinite gain margin.

The "cost function" to be minimized is the following:

$$J = \int_0^{\infty} \{ z^T(t)\ Q\ z(t) + u^T(t)\ R\ u(t) \}\ dt \qquad (II.2)$$

In Figure II.4 we have the schematic representation of the CONTROLLER.

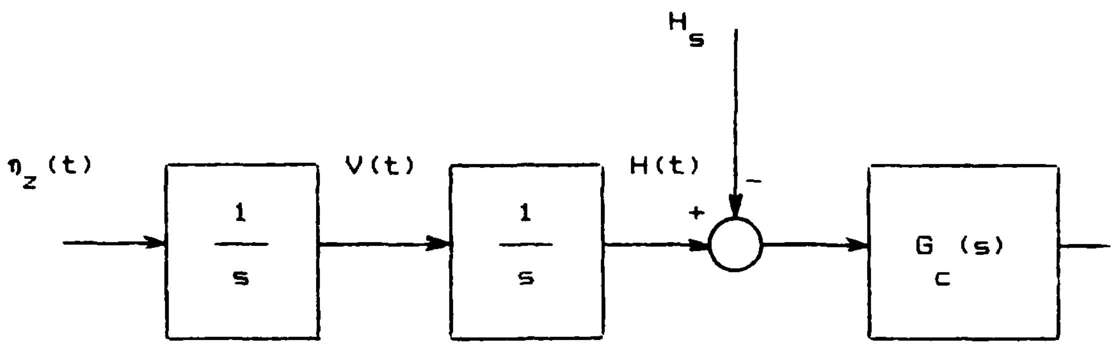

Figure II.4 - The Controller

The LEAD compensator indicated in the controller is used in order to improve the transient response characteristics. Its transfer function is :

$$G_c(s) = \frac{K*(s+w_z)}{(s+w_p)}$$

where $K = 10.0$

$w_z = 0.5$ rad/sec

$w_p = 50.0$ rad/sec

In order to have the state equations of the CONTROLLER, we draw in FIGURE II.5 its signal flow graph using the adequate transformation of the compensator and showing the states that will be feedback.

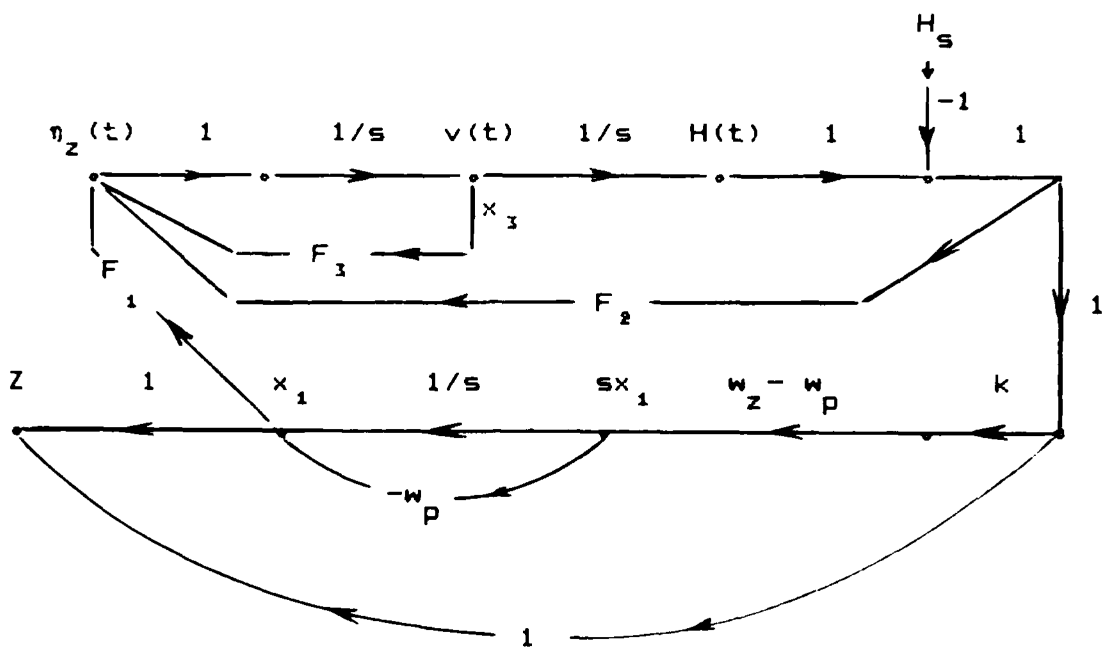

Figure II.5 Signal Flow Graph of the Controller

The state vector is defined as:

$$x(t) = [\ x_c(t) \quad (H(t)-H_s) \quad v(t)\ ]^T \tag{II.2}$$

In the Figure II.5 the states of the controller are represented as indicated in Table I.

From the diagram, we have the following STATE EQUATIONS:

$$\dot{x} = A\,x + B\,u \tag{II.3}$$

$$z = D\,x$$

The state matrices are:

$$A = \begin{bmatrix} w_p & K(w_z - w_p) & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$D = [\ 1 \ K \ 0\ ]$$

Since we consider to feedback all the states, let in equation II.1

$$Q = I \text{ (Identity)}$$

and $R = 0.04$

Using the OPTSYS program to solve that OPTIMUM CONTROL problem, the following feedback gains were obtained;

$$F_1 = 0.049$$

$$F_2 = 0.98615$$

$$F_3 = 1.4044$$

correspondents to the feedback of the states indicated in Figure II.5.

C. THE AUTOPILOT/AIRFRAME

The controller was developed assuming the transfer function of the Autopilot/Airframe as unity.This assumption implies that the achieved acceleration is equal to the commanded acceleration; however, they are different due to the aerodynamic of the missile.In order to take that in consideration we have to introduce the autopilot/airframe model.

Normally, the sea-skimming is a SKID-TO-TURN missile (turns instantaneously), but due to problems of classification we have used for our Sea--Skimming model the Autopilot given by Arrows [Ref.1] for a BANK-TO--TURN missile (has to bank in order to turn) that was designed to fly at 30,000.0 ft instead of sea level.

Take in consideration that the main objective of this work is to establish a procedure for sensitivity and robustness analysis and that we are studying only the vertical movement of the missile uncoupled of any other movement,we consider that the approximation is useful.

In Figure II.6 we have the block diagram of the missile, representing the plant without the controller.

The state equations derived from this diagram are on the Appendix A.

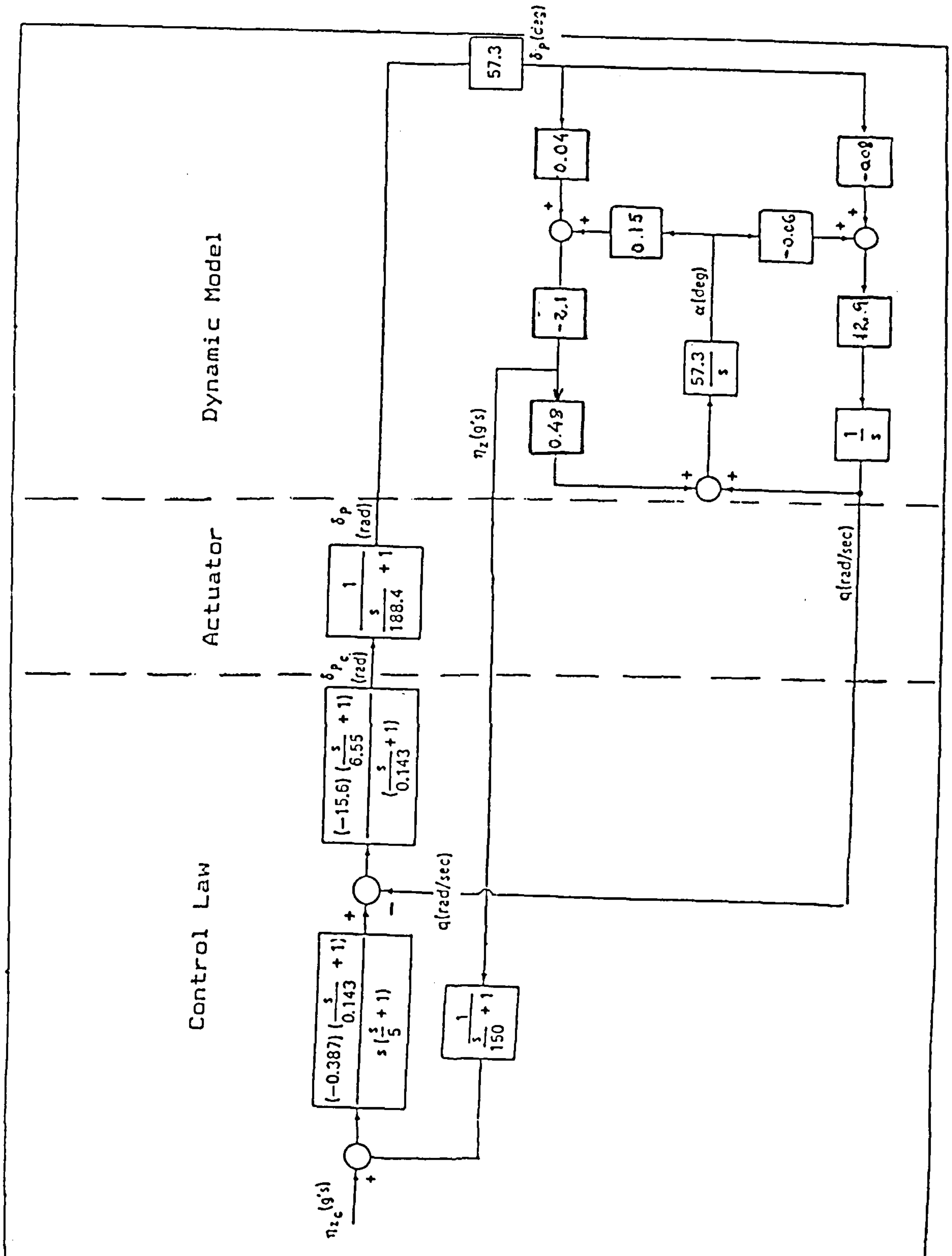

Figure II.6 Block Diagram of the Airframe/Autopilot

D. THE TIME RESPONSE

Using the software CONTROLS, the system represented by equations (II.1) was simulated and the time response for the altitude, velocity, acceleration and angle of attack were obtained.

The altitude is in Figure II.7 and we can see that the missile arrives at the desired altitude in about 10 seconds.

No overshoot is shown and a small steady-state error is presented characterizing our Type 0 system.The steady-state error was less than 5%.

The angle-of-attack ,reproduced in Figure II.8, varies between 3.7 deg and -1.2 deg satifying the condition required by the airframe, as indicated in [Refs. 1 and 2].

The acceleration is in Figure II.9 and the velocity is in Figure II.10 representing a smooth behavior of the system.

In the next chapter will be presented the Sensitivity Analysis using the developed model.

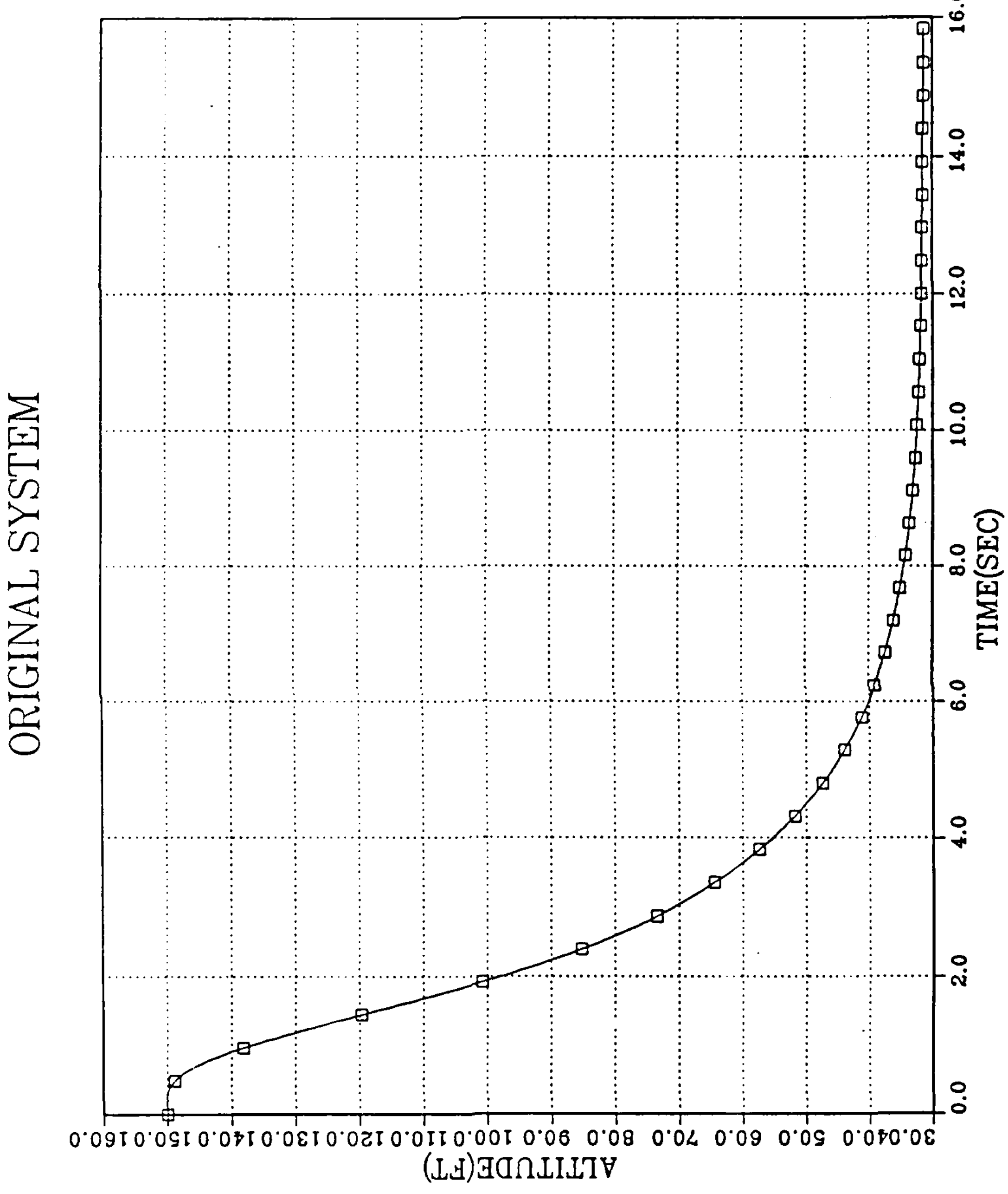

Figure II.7 - Altitude(ft) vs. Time(sec)

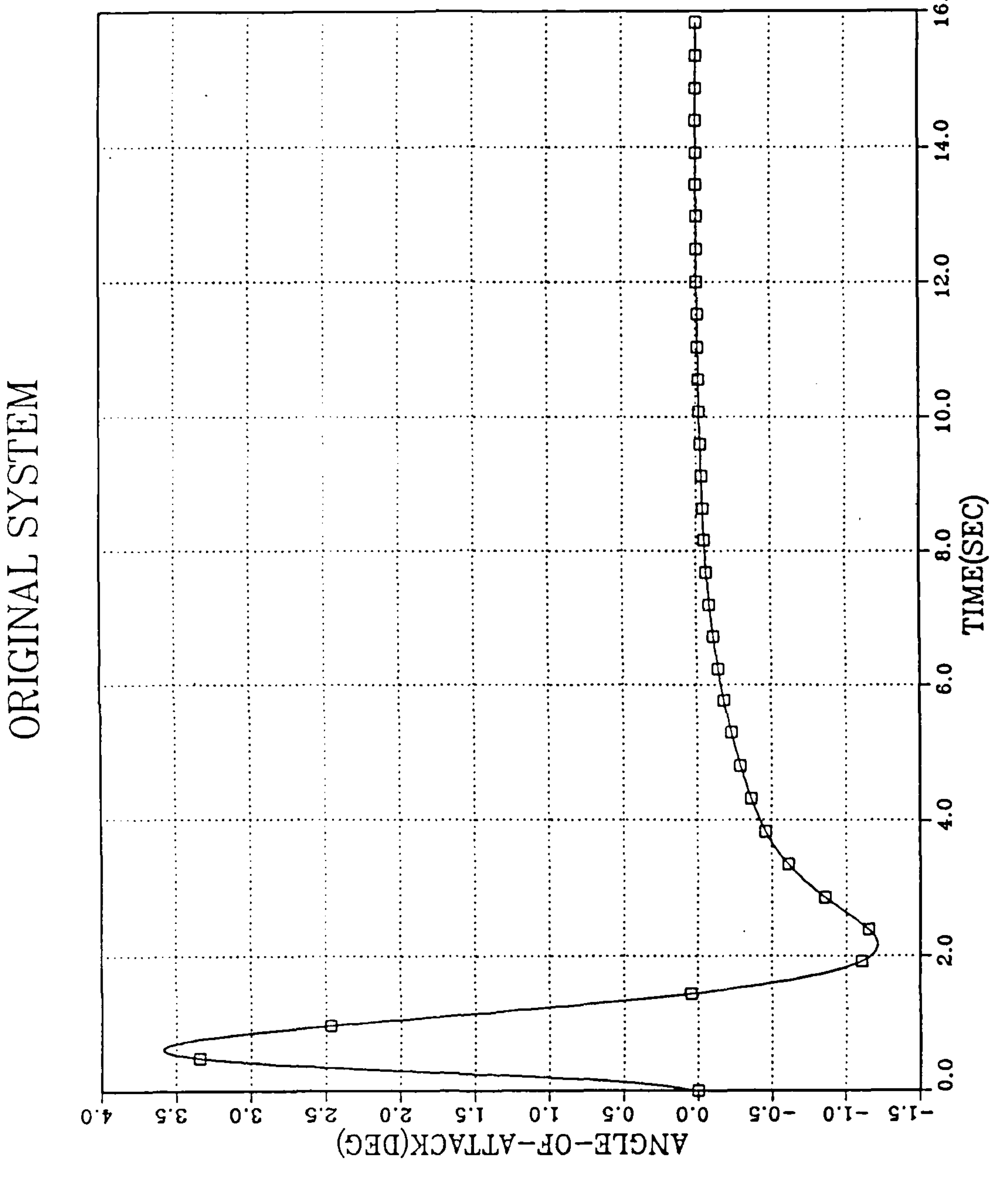

Figure II.8 - Angle-of-Attack(deg) vs. Time(sec)

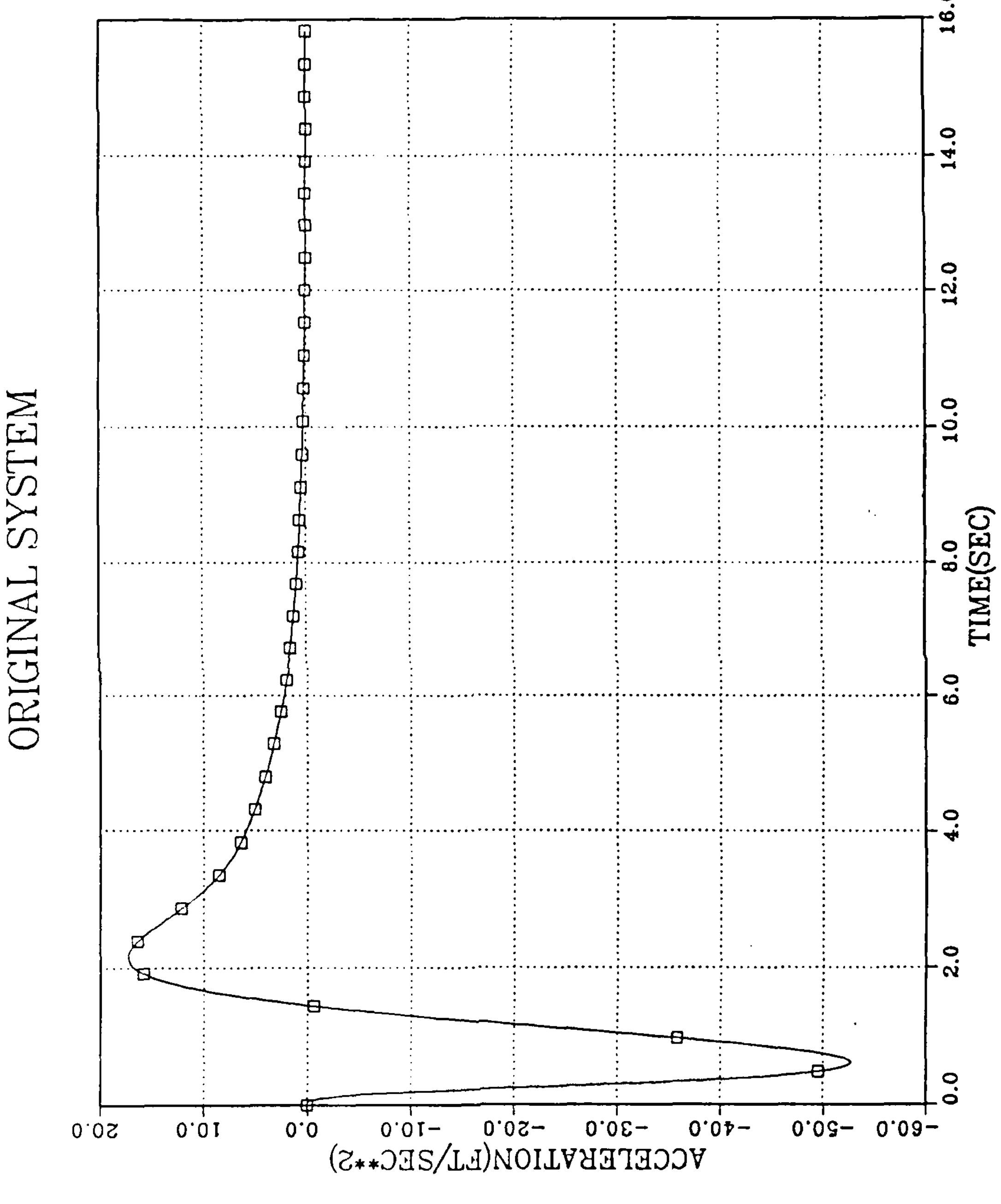

Figure II.9 - Acceleration(ft/sec^2) vs. Time(sec)

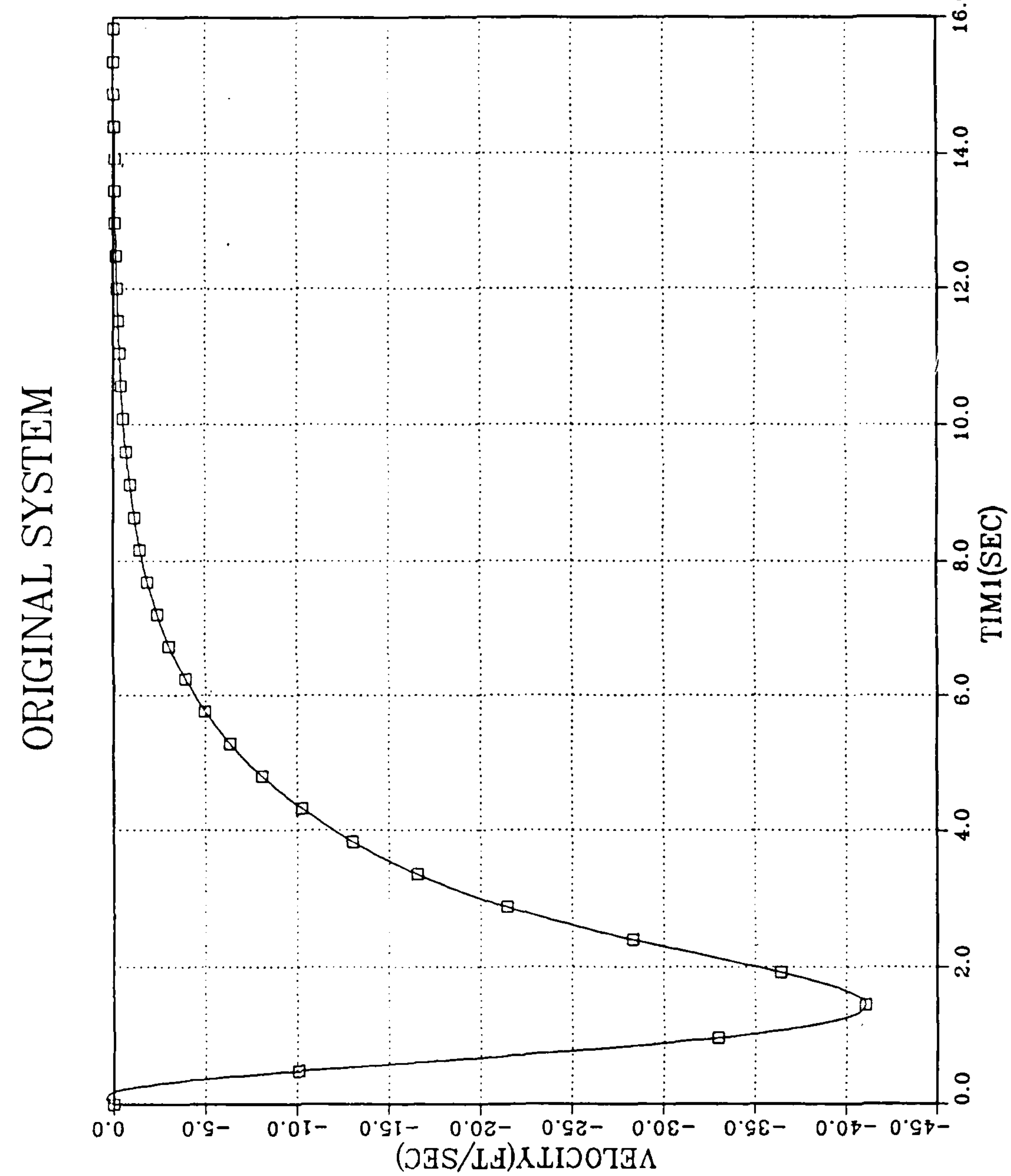

Figure II.10 - Velocity(ft/sec) vs. Time(sec)

III. SENSITIVITY ANALYSIS

A. INTRODUCTION

The objective of the SENSITIVITY ANALYSIS is to analyse how the behavior of the system is affected by variations of the aerodynamic parameters.Those variations are due to uncertainty of the design and/or problems ocurred during the flight.

From the developed model we can see that the important parameters to our system are $C_{m\alpha}$, $C_{N\alpha}$, $C_{m\delta}$ and $C_{N\delta}$, as described on TABLE I.

The variations could be interpreted as perturbations in our STATE EQUATION SYSTEM (A-matrix) and the analysis, basically, take into consideration the change in the poles positions due to change in the parameters.

As we have feedback, we replace the A matrix by the augmented A-matrix($A_a = A - B\ F$).

The sensitivity of a matrix where the elements are time-invariant,can be verified in different ways. Among those procedures, some are enumerated below.

First, using the method introduced by Golub and Van Loan [Ref.3] we can analyse the matrix to verify how it is "ill-conditioned"; where an "ill-conditioned" matrix means a very sensitive system.

In order to accomplish the analysis, we have to calculate the "condition number". It is defined as:

$$K(A_a) = || A_a || \; || A_a^{-1} ||$$

where

$|| A_a ||$ is the norm of augmented A and

$|| A_a^{-1} ||$ is the norm of inverse augmented A

The condition mumber depends upon the considered norm but it can be proven that the condition of the matrix will be the same for all norms.

When we use the 2-norm one has

$$K \; (A\;) = \frac{\bar{\sigma} \; (A_a)}{\underline{\sigma} \; (A_a)} \qquad \text{(III.1)}$$

where $\bar{\sigma}$ (A_a) and $\underline{\sigma}$ (A_a) are the maximum and minimum singular value of the A-augmented matrix, respectively.

Calculating the condition number for the different values of the parameters it is possible to establish how the sensitivity of the system changes when there are changes in some elements of the state matrix.

The problem of this kind of analysis is that we can not verify the sensitivity of the system with respect to a specified element of the state matrix but only the sensitivity of the overall system.

In Frank [Ref.4], we have another method where the analysis is done based upon the STATE SENSITIVITY EQUATIONS that could be characterized in time domain, frequency domain orin terms of a performance index.

The basic idea is to define "sensitivity functions" and analyse the behavior of the system by those functions.

The analysis starts with the system defined by:

$$\underline{\mathring{x}} = f(\underline{x}, t, \underline{u}, \underline{\alpha}) \tag{III.2}$$

where:

- $\underline{x}$ represents the state vector;
- t is time ;
- $\underline{u}$ is the input vector ; and
- $\underline{\alpha}$ represents a vactor of the different parameters.

If $\underline{\alpha}$ *is changed by* $\Delta\alpha$ *(some perturbation)* and using Taylor's expansion,we have:

$$\Delta x \ (t,\alpha) = \sum \frac{\partial x}{\partial \alpha_j} \Big|_{\alpha_0} \Delta \alpha_j \tag{III.3}$$

The subscript α_0 indicates that the partial derivative is taken at the nominal parameter values.

The partial derivative is called the trajectory vector ($\underline{\lambda}$) with respect to the $j^{\underline{th}}$ parameter and has the same dimension as the state vector.

The components of that vector are defined as the TRAJECTORY SENSITIVITY FUNCTION.

$$\lambda_{ij}(t,\underline{\alpha}_0) \triangleq \frac{\partial x(t,\alpha)}{\partial \alpha_j} \Big|_{\alpha_0} \tag{III.4}$$

The equation III.4 represents the partial derivative of the $i^{\underline{th}}$ state with respect to the $j^{\underline{th}}$ parameter. Assuming we have n states and m parameters, it will be possible to form a n x m matrix called TRAJECTORY SENSITIVITY MATRIX (III.5).

The matrix is also called the JACOBIAN MATRIX.

$$\lambda = \begin{bmatrix} \frac{\partial x1}{\partial \alpha_1} & \cdots\cdots & \frac{\partial x1}{\partial \alpha_m} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \frac{\partial xn}{\partial \alpha_1} & & \frac{\partial xn}{\partial \alpha_m} \end{bmatrix} \qquad \text{(III.5)}$$

The change in the behavior of the system will be determined by:

$$\Delta \underline{x} = \underline{\lambda} \ (t, \underline{\alpha}_0) \ \Delta \underline{\alpha} = \sum_{j=1}^{r} \lambda_j \ \Delta \alpha_j \qquad \text{(III.6)}$$

The analysis is based in how the sensitivity function changes when we change the parameters. The result could be visualized by plotes of λ as function of time, as states of the expanded system.

Finally, we consider the EIGENVALUE SENSITIVITY presented by Porter & Crossley [Ref.5].

This latter procedure was choosen to be used in the analysis of this thesis, where the sensitivity of the sistem is related to the variation of the eigenvalues of the augmented matrix ($A_a = A - B F$) with respect to the variation of the parameters.

In order to further develop our analysis, the concept of MODAL ANALYSIS will be introduced.

B. THE MODAL ANALYSIS

The concept is based in generating the input vector of a system by linear feedback of the state vector in such way that the prescribed eigenvalues are associated with the dynamic model of the resulting close-loop system.

As an example, let us assume a scalar system:

$$\dot{x} = a\ x(t) + b\ u(t); \qquad (III.7)$$

if $\quad u(t) = 0 \Longrightarrow x = x_0 \exp\ (at)$

where exp (at) defines the "mode" of the system.

If $a \leq 0$ the system is stable and if $a \geq 0$ the system is unstable.

Assuming g as the feedback gain, we have:

$$u(t) = g\ x(t) \qquad (III.8)$$

Inserting the value of u from equation III.8 in equation III.7:

$$\dot{x} = (a + b\ g)\ x(t) \qquad (III.9)$$

and, solving for x(t):

$$x(t) = x(0) \exp[(a + bg)t]$$

Now the mode of the system is defined by the exp(a+bg).

When we have a state vector, the system is represented by

$$\dot{x} = A\,x + B\,u \qquad \text{(III.10)}$$

Considering

$$u = -F\,x$$

and assuming all the states as observable and controllable, the substituting value of u in equation (III.10), we have

$$\dot{x} = (A - B\,F)\,x \quad \text{or} \quad \dot{x} = A_a\,x \qquad \text{(III.11)}$$

The precise nature of the free motion of the continuous time system following any disturbance can be described in terms of eigenvalues and eigenvectors of the augmented plant matrix (A_a).

Assuming A_a has n distinct eigenvalues $(\lambda_1, \lambda_2, \ldots, \lambda_n)$; then it also has n corresponding linearly independent eigenvectors $u_1, u_2, \ldots, u_n$ such that

$$A_a\,u_i = \lambda_i\,u_i; \; i=1,2,\ldots,n \qquad \text{(III.12)}$$

Using the calculated eigenvalues and eigenvectors we can write the modal matrix of A_a as:

$$U = [u_1\; u_2\; \ldots\; u_n]$$

where each column of U is the eigenvectors of A_a.

Based in the modal matrix U, the equation (III.12) could be written as

$$A_a U = \Lambda U \qquad \text{(III.13)}$$

with Λ as a diagonal matrix of the eigenvalues.

The matrix Λ is obtained from equation (III.13):

$$U^{-1} A_a U = \Lambda \qquad \text{(III.14)}$$

In addition to the eigenproperties of A_a, the corresponding properties of its transposed matrix (A_a') play an important role in the modal analysis.

The matrix A_a' has the same eigenvalues of A_a, but a different sets of eigenvectors.It can be represented as

$$A_a' v_j = \lambda_j v_j; \quad j = 1,2,\ldots,n$$

where v_j represents the eigenvectors of A_a'.

In [Ref.5] it is shown that

$$v_j' u_i = u_i' v_j = \delta_{ij} \qquad \text{(III.16)}$$

where δ_{ij} is the Kronecker delta; i.e.

$$\delta_{ij} = 1 \qquad \text{if} \quad i = j$$

and $$\delta_{ij} = 0 \qquad \text{if} \quad i \neq j$$

Thus in matrix form

$$V'U = I \qquad \text{(III.17)}$$

where the collumns of V are the vectors v_i.

Using the modal matrix of A_a and introducing a new state vector $\underline{y}(t)$, we have:

$$x(t) = U\, y(t) \qquad \text{(III.18)}$$

The derivative of both sides, gives us:

$$\dot{x}(t) = U\,\dot{y}(t) \qquad \text{(III.19)}$$

with $$\dot{x}(t) = A_a x(t) \qquad \text{(III.20)}$$

Combining equations (III.18),(III.19) and (III.20), we have:

$$\dot{y} = U^{-1} A_a U\, y(t)$$

and using the equation (III.14),

$$\dot{y}(t) = \Lambda\, y(t) \qquad \text{(III.21)}$$

Because the matrix Λ is diagonal the equations are uncoupled and the solutions of the system represented by (III.16) are given by

$$y_i(t) = y_i(0)\, \exp(\lambda_i t) \qquad \text{(III.22)}$$

Aplying equation III.18,

$$x_i(t) = [u_1\ u_2\ \ldots\ u_n] \begin{bmatrix} y_1 \\ y_2 \\ \cdot \\ \cdot \\ \cdot \\ y_n \end{bmatrix} \qquad \text{(III.23)}$$

which implies that

$$x(t) = u_1 y_1(0)\, \exp(\lambda_1 t) + u_2 y_2(0)\, \exp(\lambda_2 t) + \ldots$$

Taking the value of y(t) in equation (III.18),

$$y(t) = U^{-1} x(t) \qquad \text{(III.24)}$$

Using equation (III.17),

$$y_i(0) = v_i'\, x(o) \qquad \text{(III.25)}$$

Combining equations (III.23) and (III.25):

$$x(t) = \sum x(0)\ u_i v_i'\ \exp(\lambda_i t) \qquad \text{(III.26)}$$

Equation III.26 shows that the system represented by (III.8) has its modes described by the eigenvectors of A_a and its transpose and eigenvalues of A_a matrix. Also, in order to have the system stable, $x(t) \longrightarrow 0$ as $t \longrightarrow \infty$, which happens if and only if

$$\mathrm{Re}\ \lambda_i < 0 \text{ , for all the values of } i$$

In the Modal Control [Ref.5] all this procedure is shown for the case when some of the eigenvalues are equal which does not apply to our system.

As the role of the eigenvalues is so important, we can verify how the sensitivity of our controll system by analysing the change in the eigenvalues of the matrix A or the augmented A for the close loop system when any element is changed.

In order to avoid calculating the new eigenvalues for each different A, the equation (III.26) can be used to find approximated values that can be analysed analytically.

C. EIGENVALUE SENSITIVITY

In our system we consider the change in the elements of the matrix due to variations in the aerodynamic coefficients.

Basically, we concerned with the eigenvalues that are changed in the direction of the right hand plane,making the system unstable or affecting the time response.

The result of the analysis could be plots or tables relating the eigenvalues with the variation of the parameters.

For each change in the parameters, we have a new matrix.

Approximations to the eigenvalues associated with the different plant matrices in the vicinity of the original one could be used without having to compute the eigenvalues of each new matrix.

We describe the approach presented in the reference [Ref.5] applying the "first order eigenvalue sensitivity" to the change of one of the parameters and doing the complete analysis with the calculated eigenvalues.

1. First-order Eigenvalue Sensitivity

From equation III.11, we have:

$$\dot{x}(t) = A_a \, x(t)$$

Let's assume

$$A_a = \left[a_{kl} \right]$$

where a_{kl} is any element of A_a.

Differentiating equation (III.12) with respect to a_{kl} indicates that

$$\frac{\partial A}{\partial a_{kl}} u_i + A \frac{\partial ui}{\partial a_{kl}} = \frac{\partial \lambda i}{\partial a_{kl}} u_i + \lambda \frac{\partial ui}{\partial a_{kl}} \qquad \text{(III.27}$$

multiplying (III.27) by v_i' then gives

$$v_i' \frac{\partial A}{\partial a_{kl}} u_i + v_i' A \frac{\partial ui}{\partial a_{kl}} = v_i' \frac{\partial \lambda i}{\partial a_{kl}} u_i + \lambda_i v_i' \frac{\partial ui}{\partial a_{kl}}$$

As v_i' A is equal to λ_i v_i' , we have

$$v_i' \frac{\partial A}{\partial a_{kl}} u_i = v_i' \frac{\partial \lambda i}{\partial a_{kl}} u_i$$

Which reduces to the set of equations

$$\frac{\partial \lambda i}{\partial a_{kl}} = v_i(k)\ u_i(l) \quad ; \ i=k=l=1,2,\dots,n \qquad \text{(III.28)}$$

Due to

$$\frac{\partial A}{\partial a_{kl}} = \partial_{ik}\ \partial_{jl}$$

In equation (III.28), $v_i(k)$ and $u_i(l)$ represent the $k^{\underline{th}}$ element of v_i and the $l^{\underline{th}}$ element of u_i, respectively.

Those coefficients may be considered as the elements of a set of n eigenvalue sensitivity matrices.

The new eigenvalues (λ_i) of the altered matrix, when the element a_{kl} is changed will be calculated as

$$\hat{\lambda}_i = \lambda_i + \frac{\partial\ \lambda_i}{\partial a_{kl}} \Delta\ a_{kl}$$

When we have more than one parameter altered, the linearity could be used and the final eigenvalue will obtained from the summation of the sensitivity coefficients multiplied by the respective change in the coefficients.

As an example of this procedure, let's consider our system, changing the value of $C_{m\delta p}$ such that

$$\Delta C_{m\delta p} = 0.02$$

When $C_{m\delta p}$ is changed as specified, the following elements of the augmented-A matrix of the state equations presented in the Chapter II are changed:

$A_a(4,6)$: $\Delta a_{46} = 14.778$

$A_a(7,6)$: $\Delta a_{76} = 5.035$

The eigenvalues and eigenvectors of the augmented A matrix and of its tranpose were calculated using the program Matlab.

a) Eigenvalue $\hat{\lambda}_1$:

$\lambda_1 = -159.75 + j\ 18.85$

$\Delta \lambda_1 = [v_1(4)\ u_1(6)]\ \Delta a_{46} + [v_1(7)\ u_1(6)]\ \Delta a_{76}$

$v_1(4) = -0.0148 - j\ 0.0039$

$v_1(7) = 1$

$u_1(6) = 0.014 - j\ 0.027$

$\hat{\lambda}_1 = -159.815 + j\ 18.85$

b) Eigenvalue $\hat{\lambda}_2$:

$\hat{\lambda}_2 = -159.815 - j\ 18.85$ (complex conjugate of $\hat{\lambda}_2$)

c) Eigenvalue $\hat{\lambda}_3$:

$\lambda_3 = -50.0$

$\Delta\lambda_3 = [v_3(4)\ u_3(6)]\ \Delta a_{46} + [v_3(7)\ u_3(6)]\ \Delta a_{76}$

$v_3(4) = 0.0003$

$v_3(7) = 0.0039$

$u_3(6) = -0.001$

$\hat{\lambda}_3 = -49.99998$

d) Eigenvalue $\hat{\lambda}_4$:

$\lambda_4 = -8.61 + j\ 7.91$

$\Delta\lambda_4 = [v_4(4)\ u_4(6)]\ \Delta a_{46} + [v_4(7)\ u_4(6)]\ \Delta a_{76}$

$v_4(4) = -0.1667 - j\ 0.1322$

$v_4(7) = 1$

$u_4(6) = -0.0334 - j\ 0.132$

$\hat{\lambda}_4 = -8.95 + j\ 7.6355$

e) Eigenvalue $\hat{\lambda}_5$:

$\hat{\lambda}_5 = -8.95 - j\ 7.6355$

f) Eigenvalue $\hat{\lambda}_6$:

$\lambda_6 \quad = -1.82 + j\ 2.02$

$\Delta\lambda_6 \quad = [v_6(4)\ u_6(6)]\ \Delta a_{46} + [v_6(7)\ u_6(6)]\ \Delta a_{76}$

$v_6(4) = 0.1718 - j\ 0.0161$

$v_6(7) = -0.5737 + j\ 0.1689$

$u_6(6) = 0.01 + j\ 0.0363$

$\hat{\lambda}_6 \quad = -1.846 + j\ 2.05$

g) Eigenvalue $\hat{\lambda}_7$:

$\hat{\lambda}_7 \quad = -1.846 - j\ 2.05$

h) Eigenvalue $\hat{\lambda}_8$:

$\lambda_8 \quad = -2.71$

$\Delta\lambda_8 \quad = [v_8(4)\ u_8(6)]\ \Delta a_{46} + [v_8(7)\ u_8(6)]\ \Delta a_{76}$

$v_8(4) = 0.1741$

$v_8(7) = -0.6597$

$u_8(6) = -0.0396$

$\hat{\lambda}_8 \quad = -2.681$

i) Eigenvalue $\hat{\lambda}_9$:

$\lambda_9 \quad = -0.5$

$\Delta\lambda_9 \quad = [v_9(4)\ u_9(6)]\ \Delta a_{46} + [v_9(7)\ u_9(6)]\ \Delta a_{76}$

$v_9(4) = 0.1718 + j\ 0.016$

$v_9(7) = -0.5737 - j\,0.1689$

$u_9(6) = 0.01 - j\,0.0363$

$\hat{\lambda}_9 = -0.4741$

j) Eigenvalue λ_{10} :

$\lambda_{10} = -0.14$

$\Delta\lambda_{10} = [v_{10}(4)\ u_{10}(6)]\ \Delta a_{46} + [v_{10}(7)\ u_{10}(6)]\ \Delta a_{76}$

$v_{10}(4) = 0.1741$

$v_{10}(7) = -0.6597$

$u_{10}(6) = -0.0396$

$\hat{\lambda}_{10} = -0.1104$

For the analysed parameter, our system is unsensitive, taking in consideration the small differences between the new and original eigenvalues.

Repeating this procedure for all different parameter values will give us the approximation of the new eigenvalues and that can be helpfull in a qualitative analysis of what parameter has more effect over our model.

As we want to verify the effect in the time response and due to the Software available the analysis will be done completely with the calculated exact values.

2. Exact Eigenvalues

Using the Controls Program, we will verify the variation of the eigenvalues of the augmented A matrix (A - FB) with respect to the variation of the parameters and by simulation we analyse the effects on the time response.

At the end of the analysis we compare the results of the example used for the first order approximation with the exact values that were calculated.

As reference, the eigenvalues of the original system are:

$\lambda_1 = -159.746 + j\ 18.979$

$\lambda_2 = -159.746 - j\ 18.979$

$\lambda_3 = -49.9992$

$\lambda_4 = -8.60697 + j\ 7.90968$

$\lambda_5 = -8.60697 - j\ 7.90968$

$\lambda_6 = -1.81854 + j\ 2.0238$

$\lambda_7 = -1.81854 - j\ 2.0238$

$\lambda_8 = -2.70266$

$\lambda_9 = -0.503819$

$\lambda_{10} = -0.143073$

Each aerodynamic coefficient, that takes part of the considered model was changed up to $\pm 25\%$ and the eigenvalues of the perturbed system matrix (A-augmented) were analysed as well as the TIME RESPONSE of the system .

a. $C_{m\alpha}$

On TABLE II we have the eigenvalues related to the considered values of $C_{m\alpha}$ and as we can see, with exception of the 7^{th} eigenvalue, the changes are small and the system is pratically insensitive to that variation.

The time responses are shown in Figures III.1, III.2,III.3 and III.4 .

- Angle-of-attack (Figure III.1) - changes in the same direction of $C_{m\alpha}$ with the minimum value changing about 0.5 degrees in the negative direction. The maximum value stays almost constant ;
- Acceleration (Figure III.2) - the change was less than 5 ft/sec^2 either the maximum or the minimum values;
- Velocity (Figure III.3) - almost the same ; and
- Altitude (Figure III.4) - almost the same.

b. $C_{m\delta p}$

The change in the eigenvalues are on TABLE III ; the time response is on Figure III.5,III.6,III.7 and III.8.

The variation of the eigenvalues are slightly higher with changes in $C_{m\delta p}$ but the system stays stable and the time response has a small variation.

- Angle-of-Attack (Figure III.5) - The minimum values increases negativelly about 0.5 degrees but in the contrary direction of the coefficient, i.e., increasing the value of $C_{m\delta p}$ negativelly, the minimum was smaller;

- Acceleration (Figure III.6) - small changes;

- Velocity (Figure III.7) - small changes;

- Altitude (Figure III.8) - pratically the same.

c. $C_{N\alpha}$

The change in the eigenvalues are on TABLE IV and the time response is on Figures III.9,III.10,III.11 and III.12.

The changes in the eigenvalues are small but the effect on the time response is more sensible, mainly on the angle-of-attack.

- Angle-of-Attack (Figure III.9) - the maximum and the minimum increase 1.0 degree in the positive and negative directions, respectivelly;

- Acceleration (Figure III.10) - maximum changes of 10% in the negative direction (minimum) and 20% in the positive direction (maximum).These values are smaller when we increase the value of $C_{N\alpha}$;

- Velocity (Figure III.11) - small changes;

- Altitude (Figure III.12) - does not change.

d. $C_{N\delta p}$

The change in the eigenvalues are on Table V and the time response are on the Figures III.13,III.14,III.15 and III.16

- Angle-of-Attack (Figure III.13) - Changes almost 1.0 degree with the maximum and minimum increasing when the value of the parameter increases;

- Acceleration (Figure III.14) - the maximum increases with the value of the parameter;

- Velocity (Figure III.15) - pratically the same for all the considered values of the parameter;

- Altitude (Figure III.16) - constant.

TABLE II

$C_{m\alpha}$ - SENSITIVITY

$C_{m\alpha}$	-0.04		-0.05	
	-159.741	+ j 18.9843	-159.743	+ j 18.9816
	-159.741	- j 18.9843	-159.743	- j 18.9816
	-49.9992		-49.9992	
	-8.55822	+ j 6.37130	-8.56386	+ j 7.18736
	-8.55822	- j 6.37130	-8.56386	- j 7.18736
	-2.31740	+ j 2.95574	-2.14117	+ j 2.43275
	-2.31740	- j 2.95574	-2.14117	- j 2.43275
	-1.79944		-2.14200	
	-0.51650		-0.510282	
	-0.14304		-0.14306	
$C_{m\alpha}$	-0.07		-0.08	
	-159.748	+ j 18.9762	-159.751	+ j 18.9735
	-159.748	- j 18.9762	-159.751	- j 18.9735
	-49.9992		-49.9992	
	-8.66197	+ j 8.55389	-8.72178	+ j 9.14021
	-8.66197	+ j 8.55389	-8.72178	+ j 9.14021
	-1.47981	+ j 1.81088	-1.22077	+ j 1.70081
	-1.47981	- j 1.81088	-1.22077	- j 1.70081
	-3.27131		-3.67125	
	-0.49779		-0.49148	
	-0.14308		-0.14308	

TABLE III

$C_{m\delta p}$ - SENSITIVITY

$C_{m\delta p}$	-0.06		-0.07	
	-162.604	+ j 16.7131	-161.200	+ j 17.9141
	-162.604	- j 16.7131	-161.200	- j 17.9141
	-49.9992		-49.9992	
	-5.74603	+ j 8.86120	-7.13033	+ j 8.50992
	-5.74603	- j 8.86120	-7.13033	- j 8.50992
	-1.24694	+ j 1.66681	-1.54230	+ j 1.80335
	-1.24694	- j 1.66681	-1.54230	- j 1.80335
	-3.86370		-3.30548	
	-0.49103		-0.49828	
	-0.14311		-0.14309	
$C_{m\delta p}$	-0.09		-0.10	
	-158.243	+ j 19.9192	-156.678	+ j 20.7492
	-158.243	- j 19.9192	-156.678	- j 20.7492
	-49.9991		-49.9990	
	-10.1621	+ j 6.98960	-11.7920	+ j 5.53519
	-10.1621	- j 6.98960	-11.7920	- j 5.53519
	-1.47981	+ j 1.81088	-2.03752	+ j 2.55411
	-1.47981	- j 1.81088	-2.03752	- j 2.55411
	-2.27254		-2.02276	
	-0.50786		-0.51120	
	-0.14307		-0.14306	

TABLE IV

$C_{N\alpha}$ - SENSITIVITY

$C_{N\alpha}$	0.13	0.14
	-159.750 + j 18.9895	-159.748 + j 18.9842
	-159.750 - j 18.9895	-159.748 - j 18.9842
	-49.9992	-49.9992
	-8.76667 + j 8.27012	-8.68784 + j 8.09494
	-8.76667 - j 8.27012	-8.68784 - j 8.09494
	-1.18629 + j 1.74321	-1.47163 + j 1.84903
	-1.18629 - j 1.74321	-1.47163 - j 1.84903
	-3.63456	-3.22775
	-0.48878	-0.49663
	-0.14314	-0.14310
$C_{N\alpha}$	0.16	0.17
	-159.744 + j 18.9736	-159.741 + j 18.9683
	-159.744 - j 18.9736	-159.741 - j 18.9683
	-49.9992	-49.9992
	-8.52441 + j 7.71274	-8.44077 + j 7.50213
	-8.52441 - j 7.71274	-8.44077 - j 7.50213
	-2.15317 + j 2.32249	-2.40105 + j 2.67918
	-2.15317 - j 2.32249	-2.40105 - j 2.67918
	-2.20610	-1.88565
	-0.51041	-0.51651
	-0.14305	-0.14303

TABLE V

$C_{N\delta p}$ - SENSITIVITY

$C_{N\delta p}$	0.02	0.03
	-159.004 + j 12.1138	-159.748 + j 18.9842
	-159.004 - j 12.1138	-159.748 - j 18.9842
	-49.9996	-49.9992
	-9.37545 + j 7.11519	-8.68784 + j 8.09494
	-9.37545 - j 7.11519	-8.68784 - j 8.09494
	-2.24432 + j 2.56109	-1.47163 + j 1.84903
	-2.24432 - j 2.56109	-1.47163 - j 1.84903
	-1.98723	-3.22775
	-0.51356	-0.49663
	-0.14304	-0.14310

$C_{N\delta p}$	0.05	0.06
	-160.106 + j 21.5632	-160.459 + j 23.8437
	-160.106 - j 21.5632	-160.459 - j 23.8437
	-49.9990	-49.9988
	-8.25204 + j 8.25703	-7.91449 + j 8.57438
	-8.25204 - j 8.25703	-7.91449 - j 8.57438
	-1.57370 + j 1.85345	-1.35009 + j 1.75196
	-1.57370 - j 1.85345	-1.35009 - j 1.75196
	-3.18783	-3.60970
	-0.49848	-0.49275
	-0.14310	-0.14312

D. CONCLUSION

As a conclusion for the sensitivity analysis, we have the following:

With respect to the eigenvalues, the pitch control system is more sensitive to changes in $C_{m\delta p}$ and $C_{N\delta p}$ than changes in $C_{m\alpha}$ and $C_{N\alpha}$; and

Considering the time response, we can verify that the angle-of-attack is sensitive to all the parameters with higher variation with $C_{N\alpha}$; the acceleration is more sensitive to $C_{m\alpha}$ and $C_{N\alpha}$; the velocity has the maximum value changed less than 10% with respect to all the parameters; and the altitude is insensible for the considered variations.

It is not necessary to emphasize the importance of this analysis for the design of the control system, but thinking only in the eigenvalues and comparing the results of the first order approximation presented for $C_{m\delta p}$ with those of Table III for $C_{m\delta p}$ changed to -0.06 ($\Delta C_{m\delta p}$= 0.02) we can have an idea of the importance of the parameter, using that approximation.

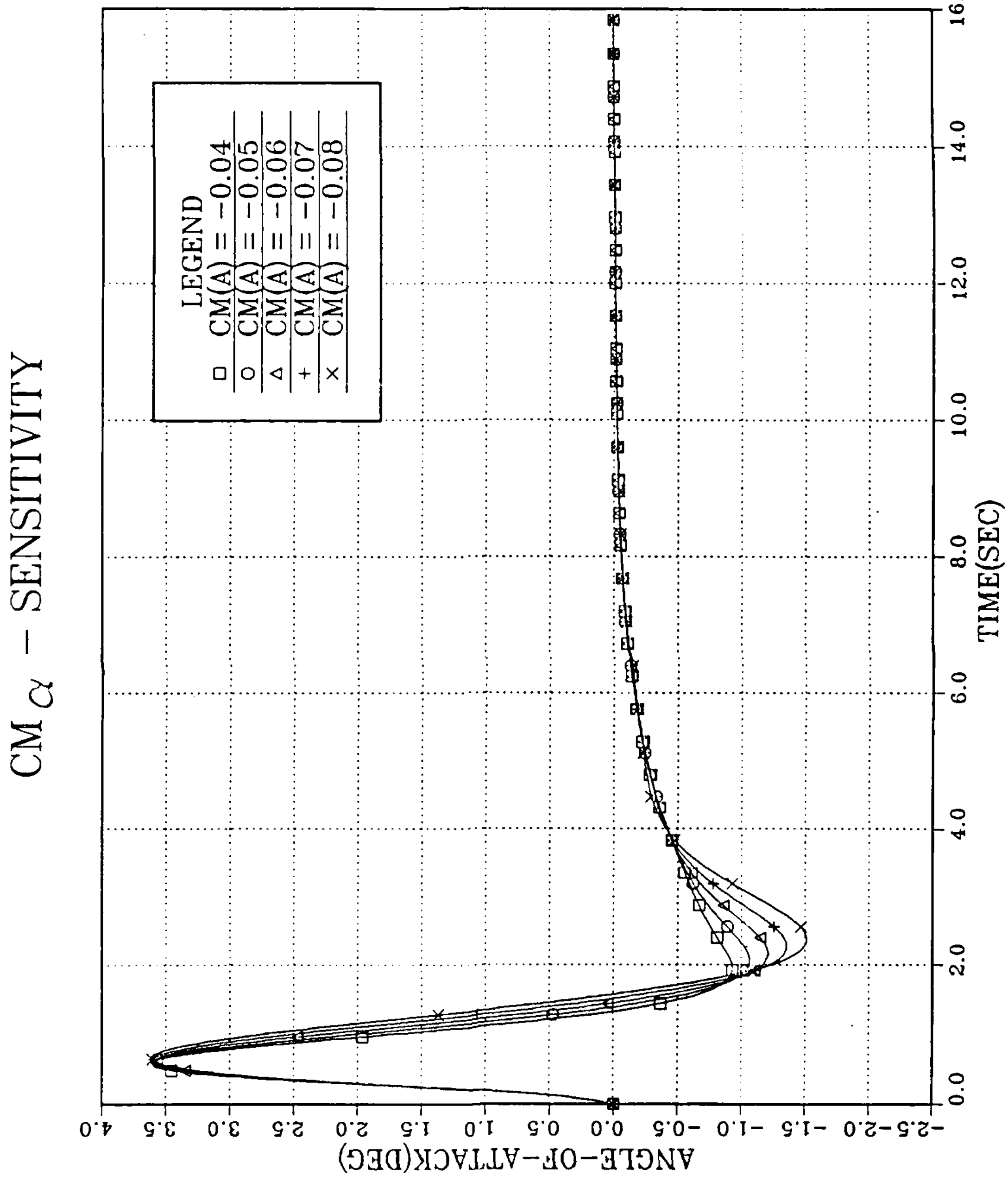

Figure III.1 Angle-of Attack vs Time for different values of $C_{m\alpha}$

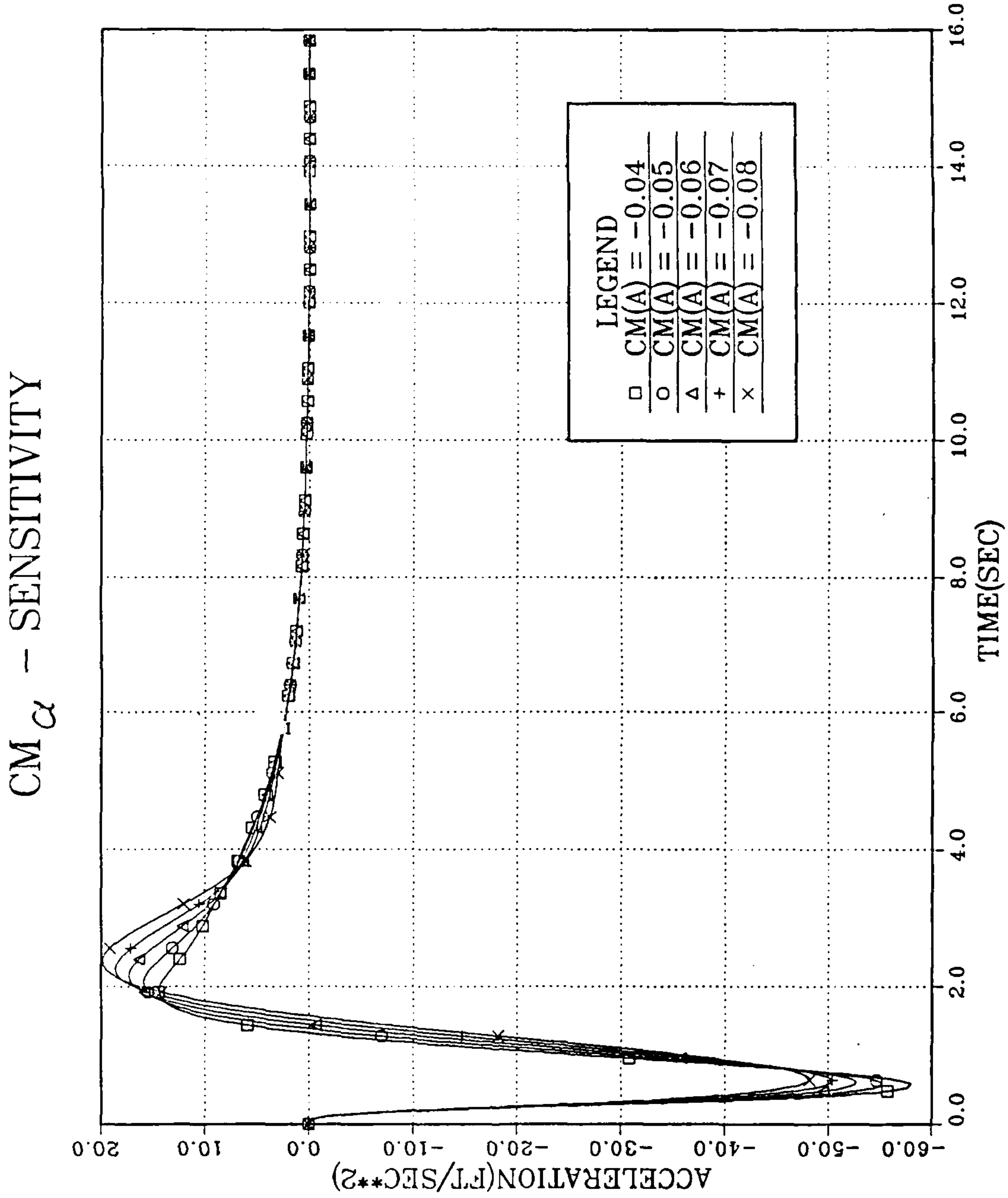

Figure III.2 Acceleration vs Time for different values of $C_{m\alpha}$

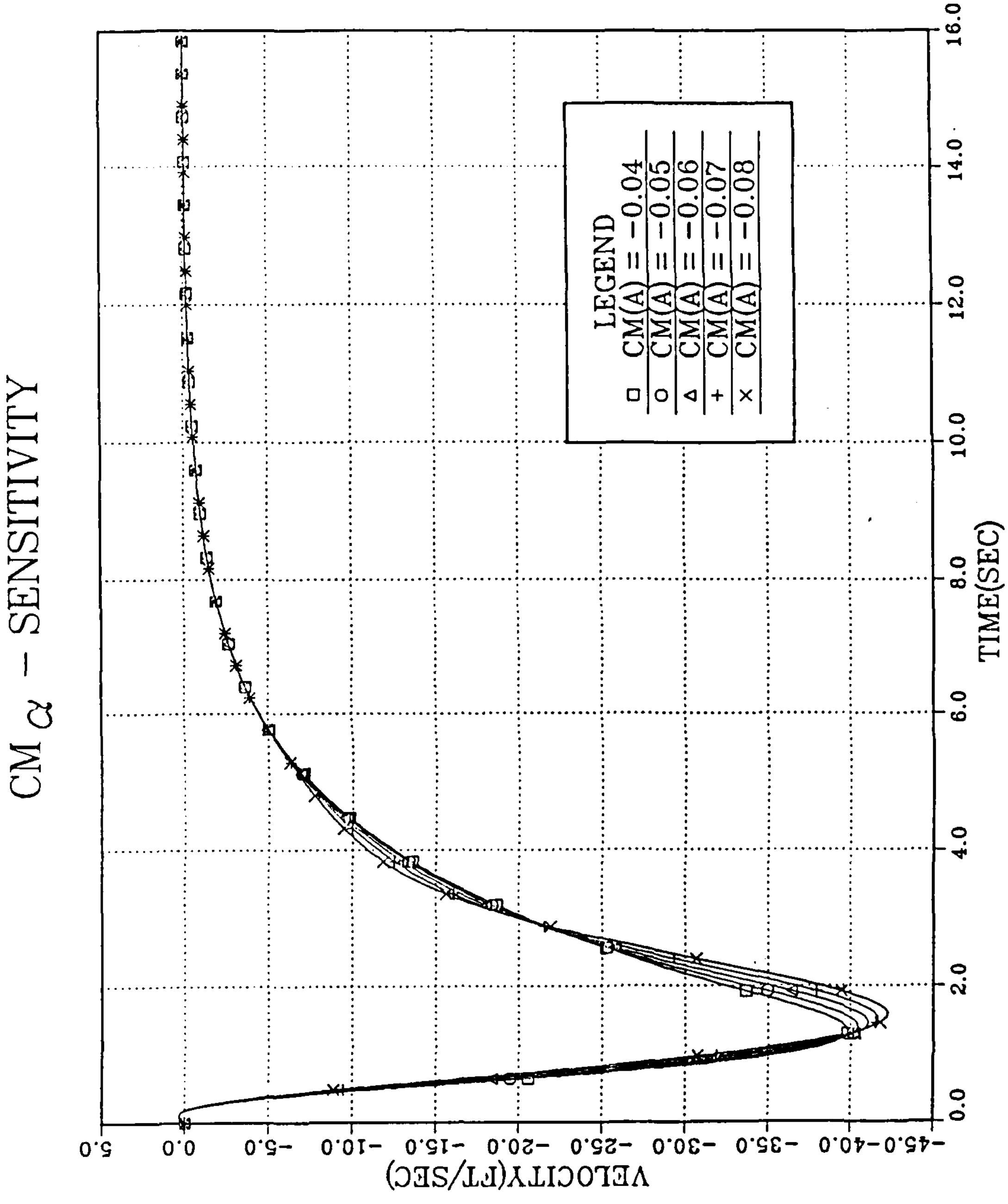

Figure III.3 Velocity vs Time
for different values of $C_{m\alpha}$

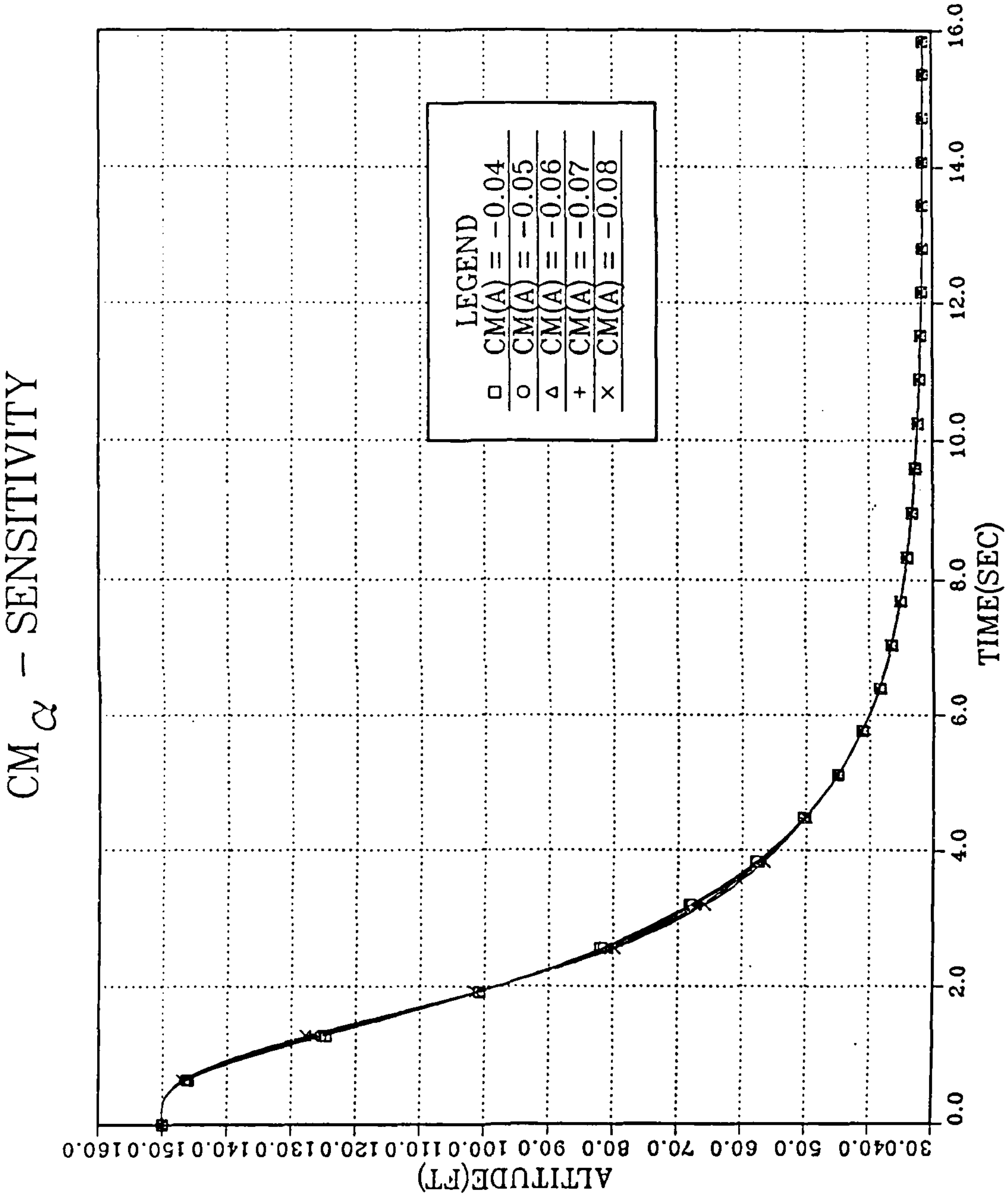

Figure III.4 Altitude vs Time
for different values of $C_{m\alpha}$

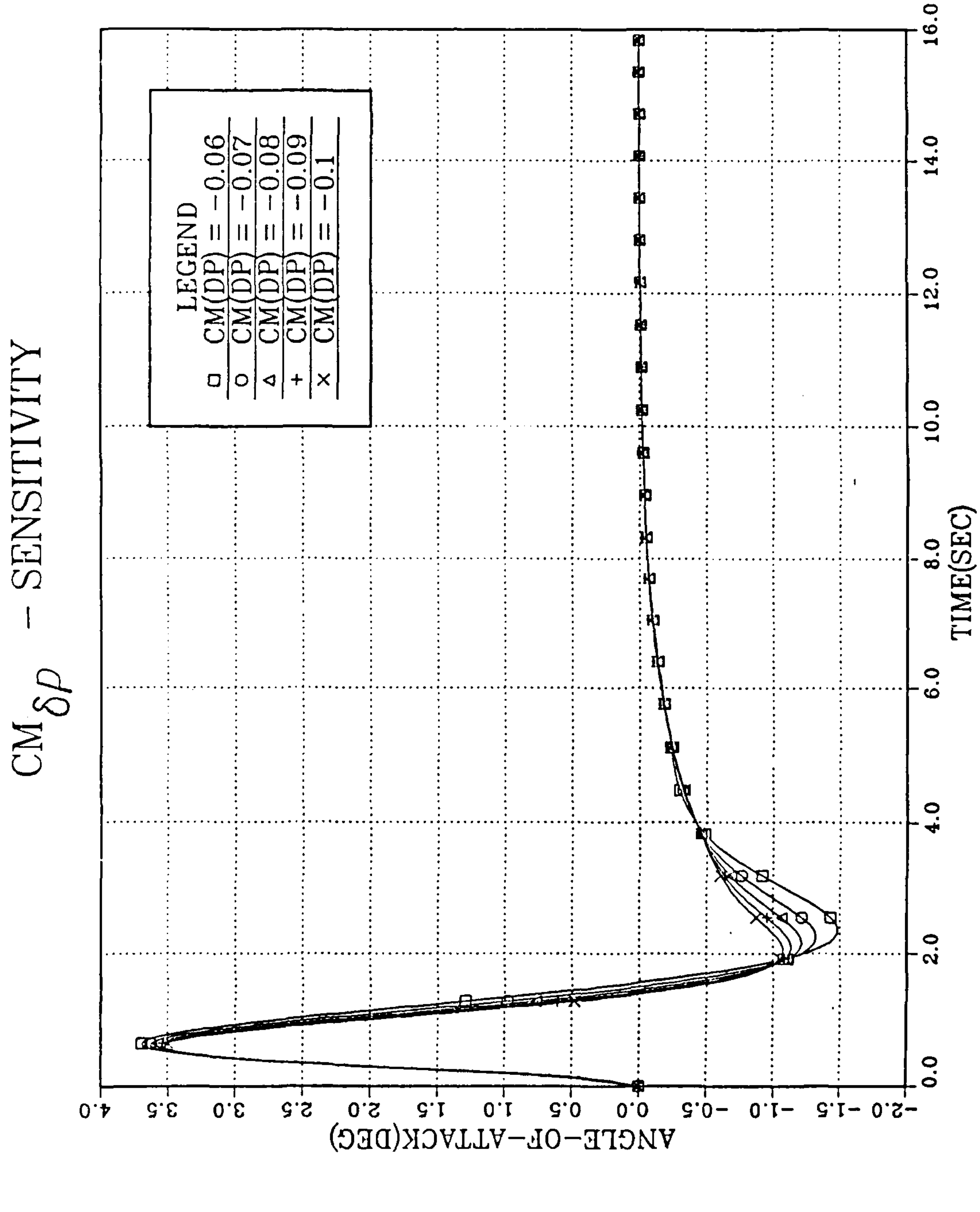

Figure III.5 Angle-of-Attack vs Time for different values of $C_{m\delta p}$

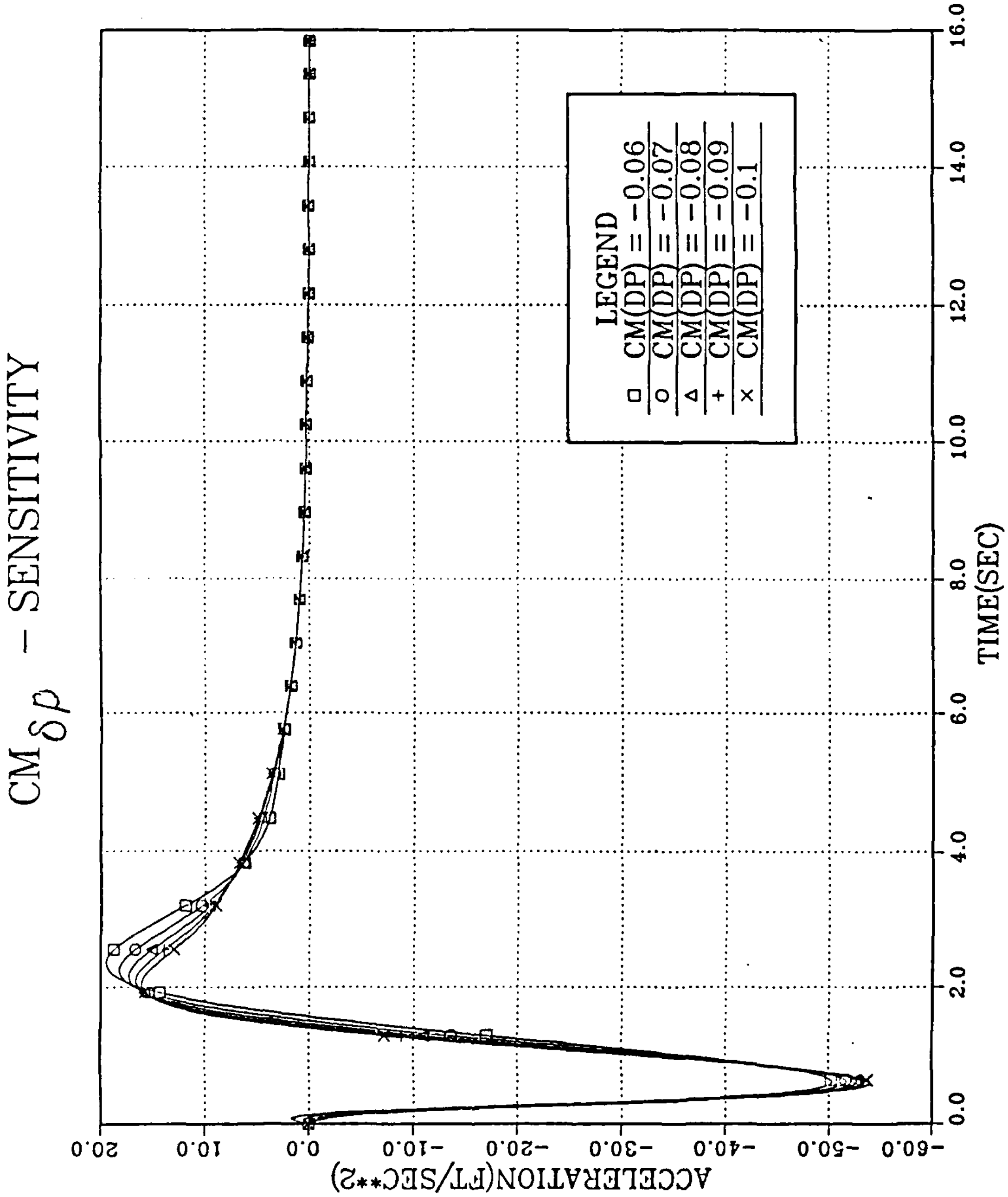

Figure III.6 Acceleration vs Time
for different values of $C_{m\delta p}$

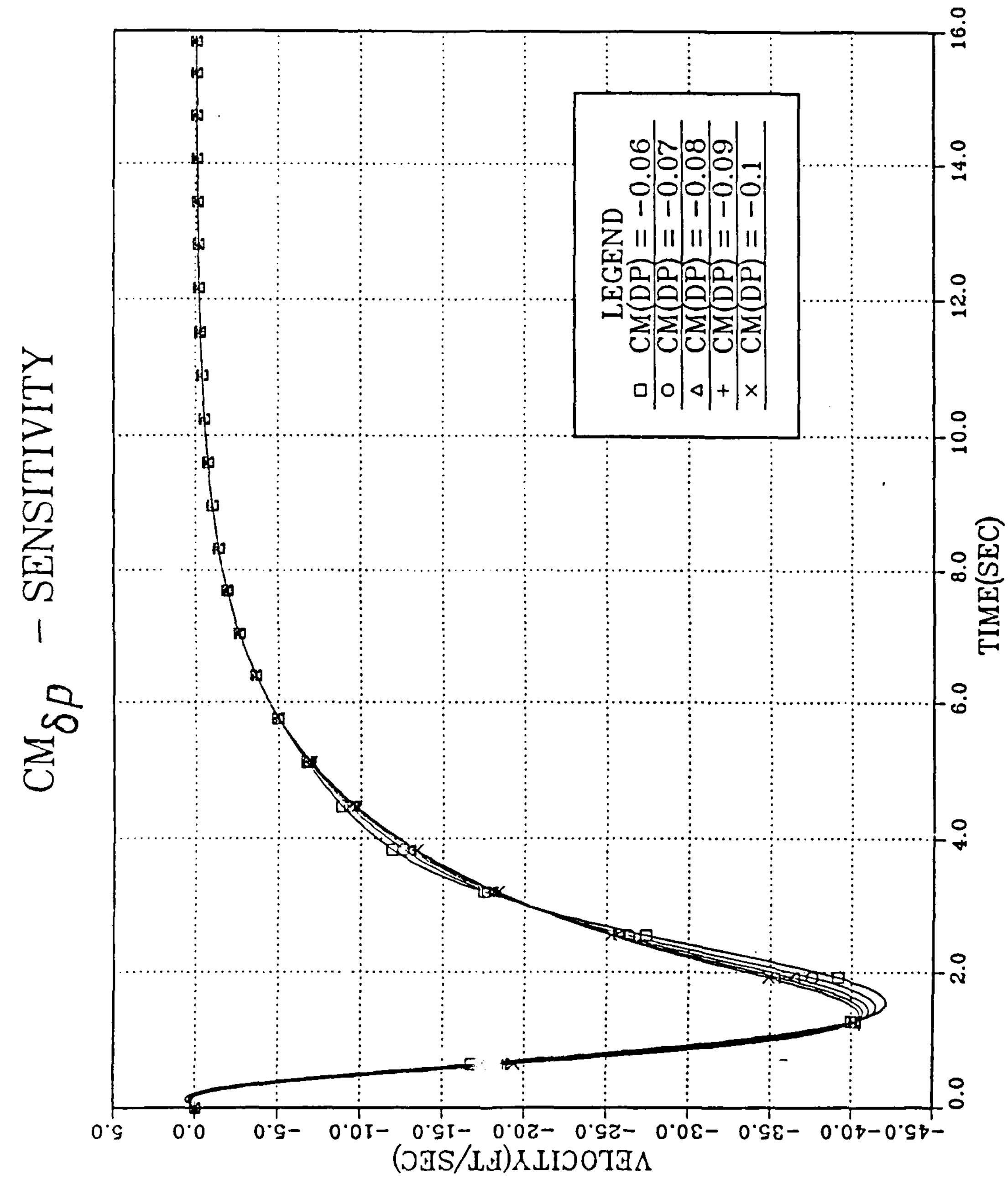

Figure III.7 Velocity vs Time
for different values of $C_{m\delta p}$

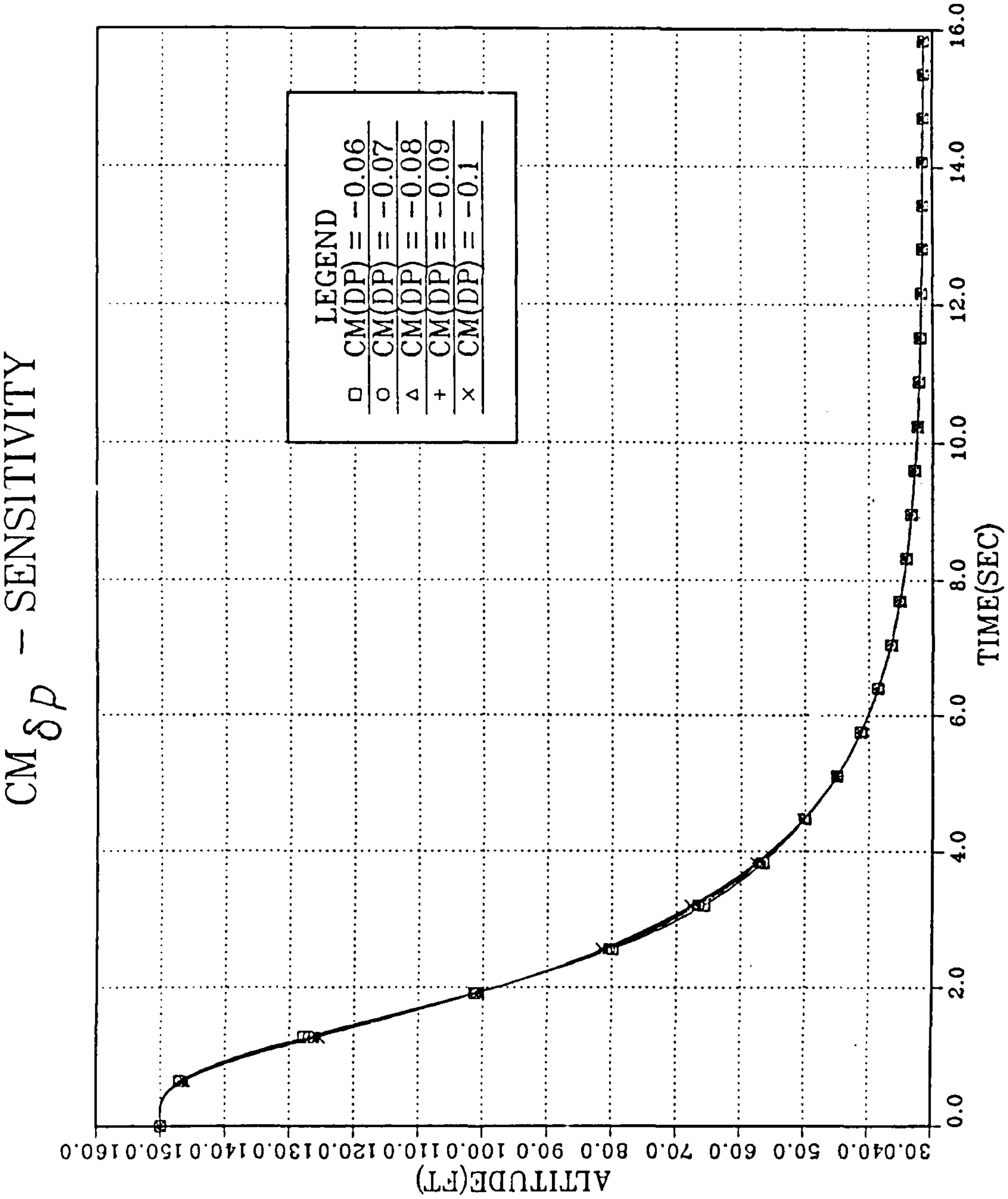

Figure III.8 Altitude vs Time
for different values of $C_{m_{\delta p}}$

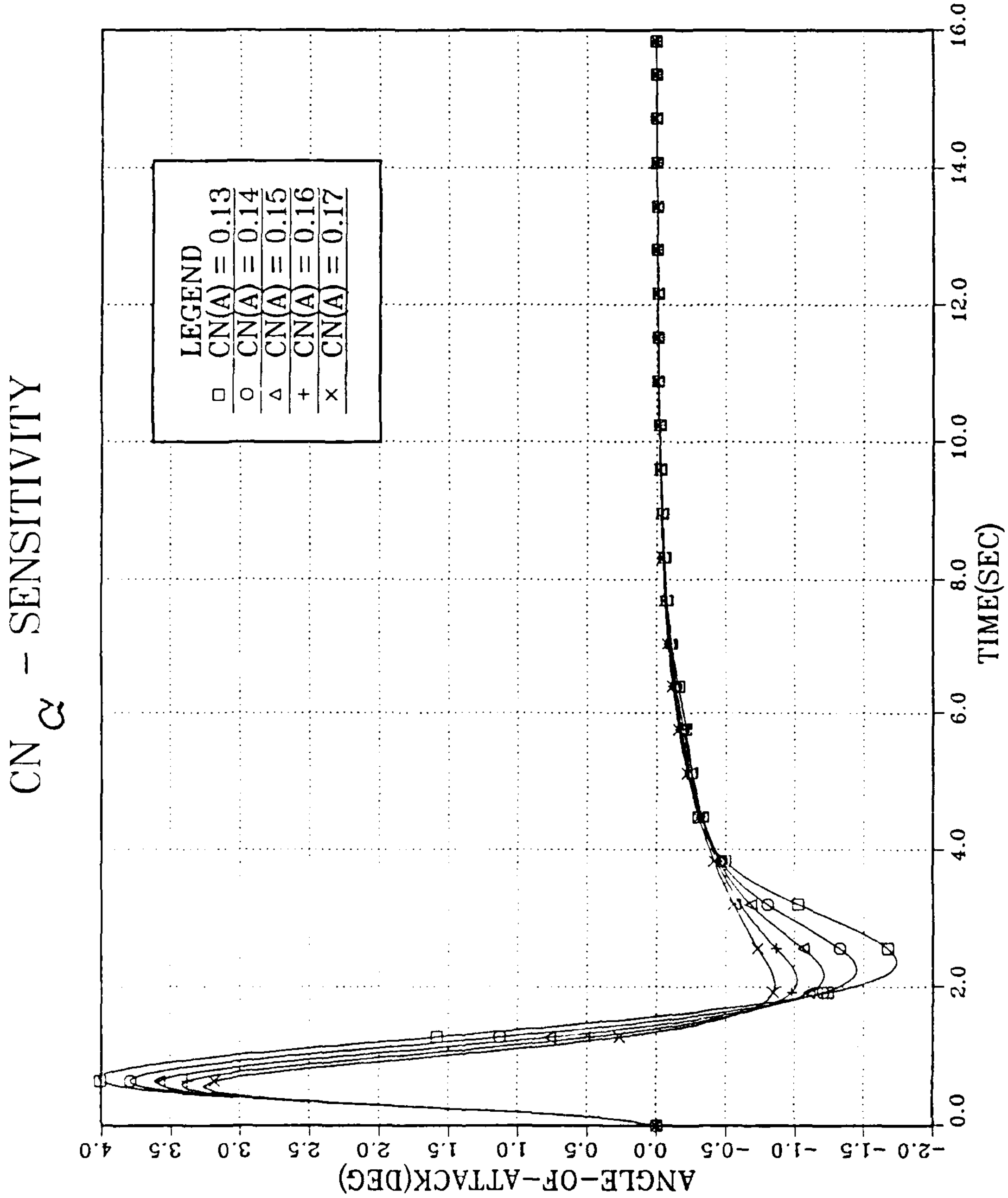

Figure III.9 Angle-of-Attack vs Time for different values of $C_{N\alpha}$

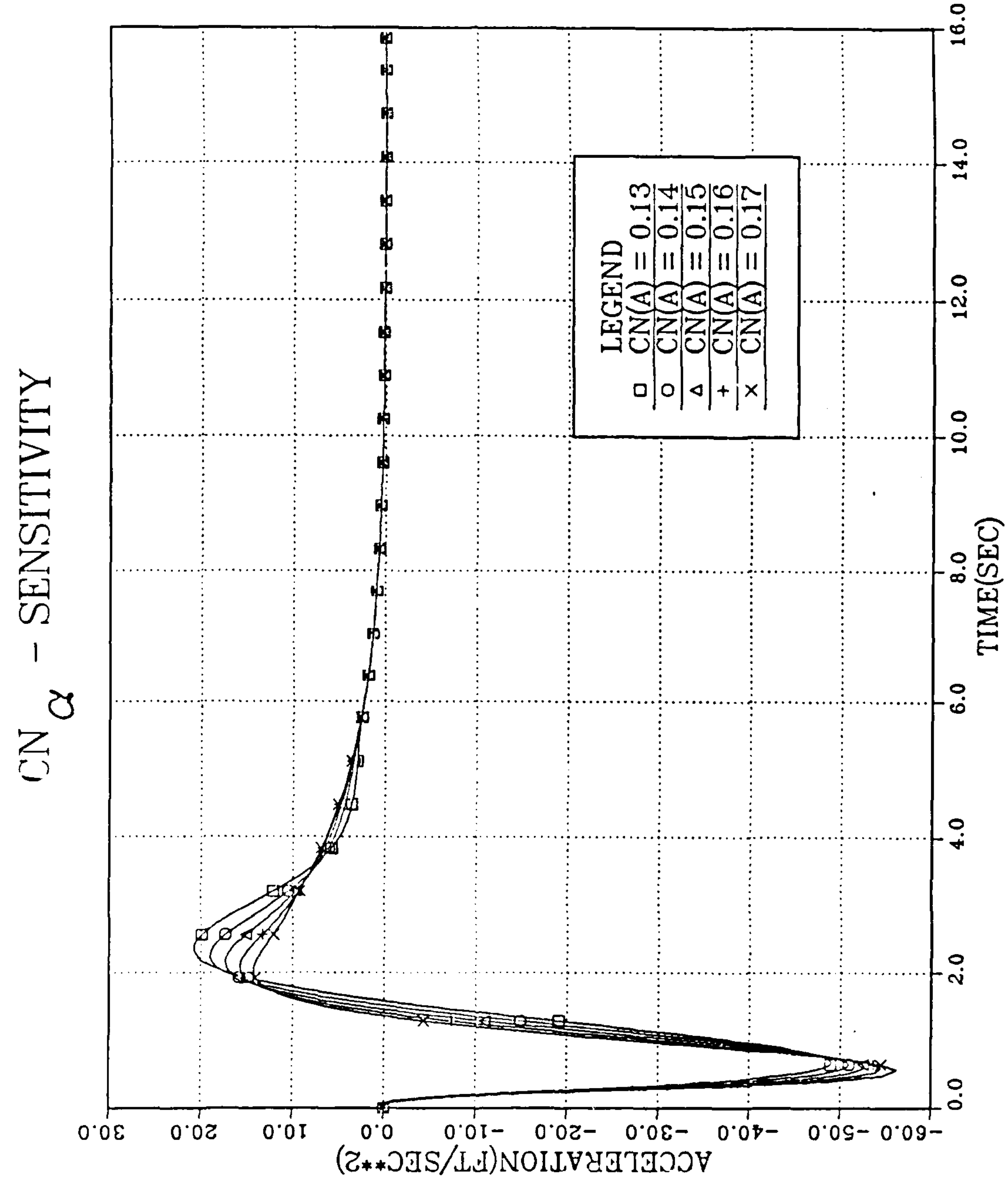

Figure III.10 Acceleration vs Time for different values of $C_{N\alpha}$

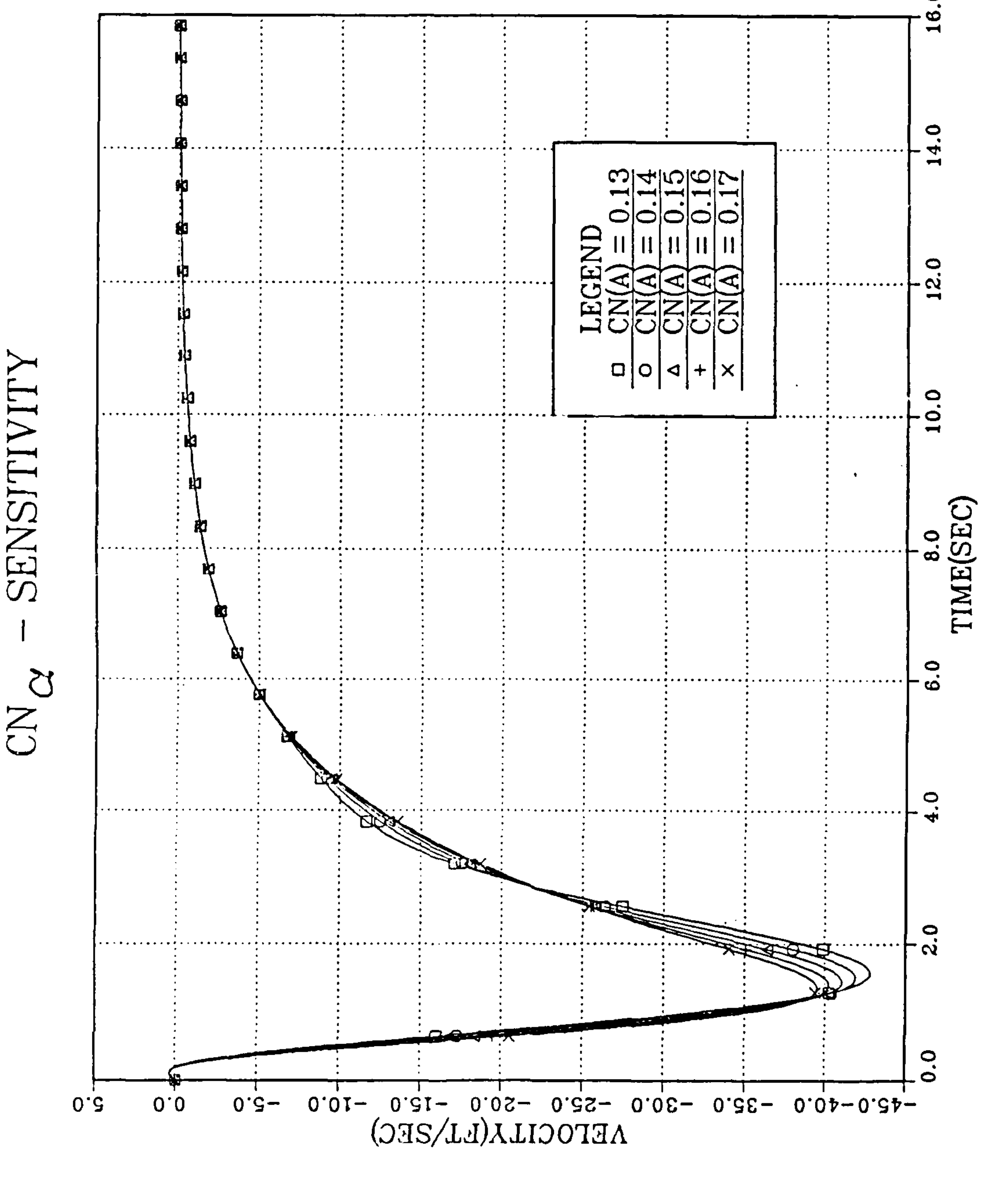

Figure III.11 Velocity vs Time
for different values of $C_{N\alpha}$

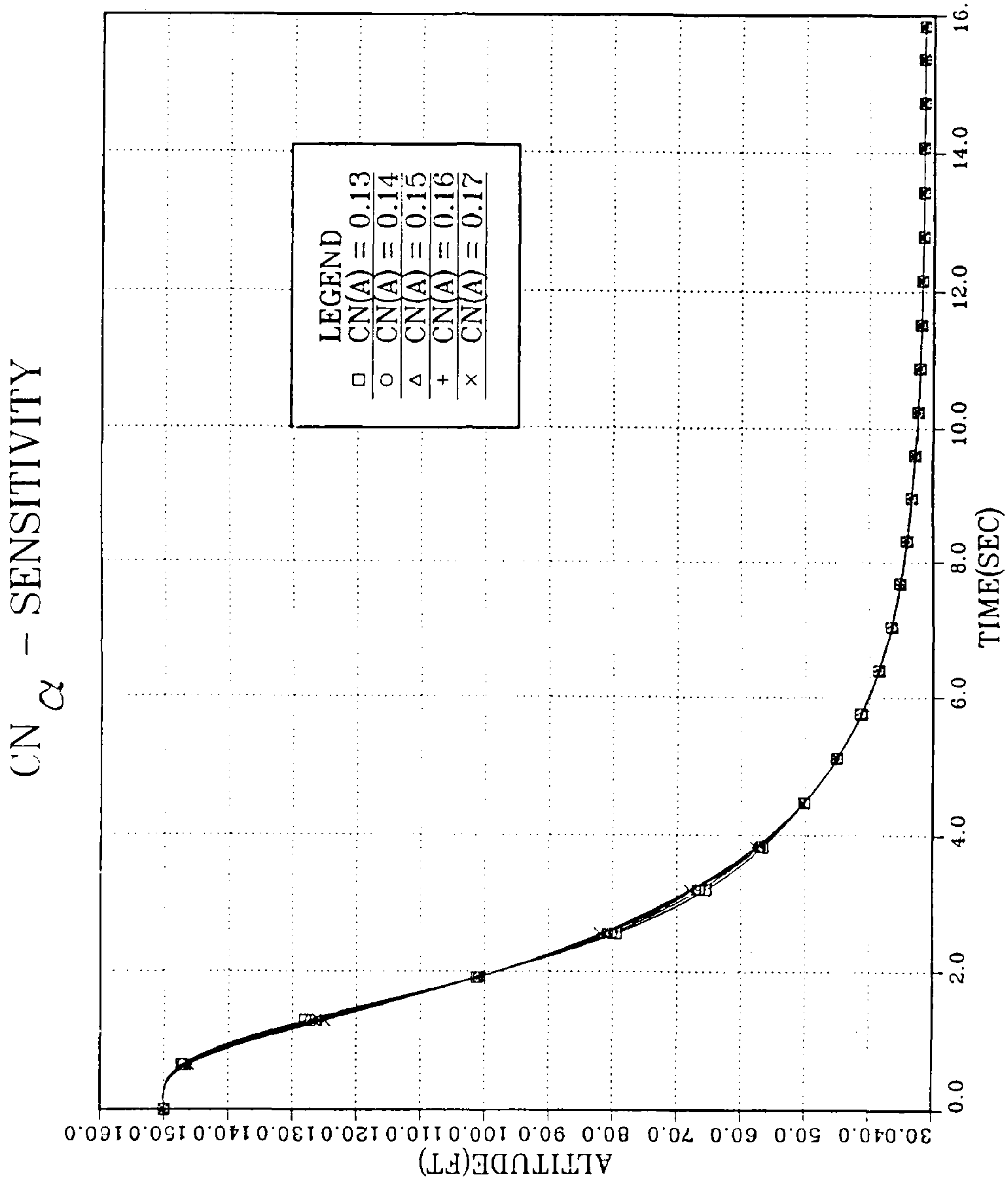

Figure III.1 Altitude vs Time
for different values of $C_{N\alpha}$

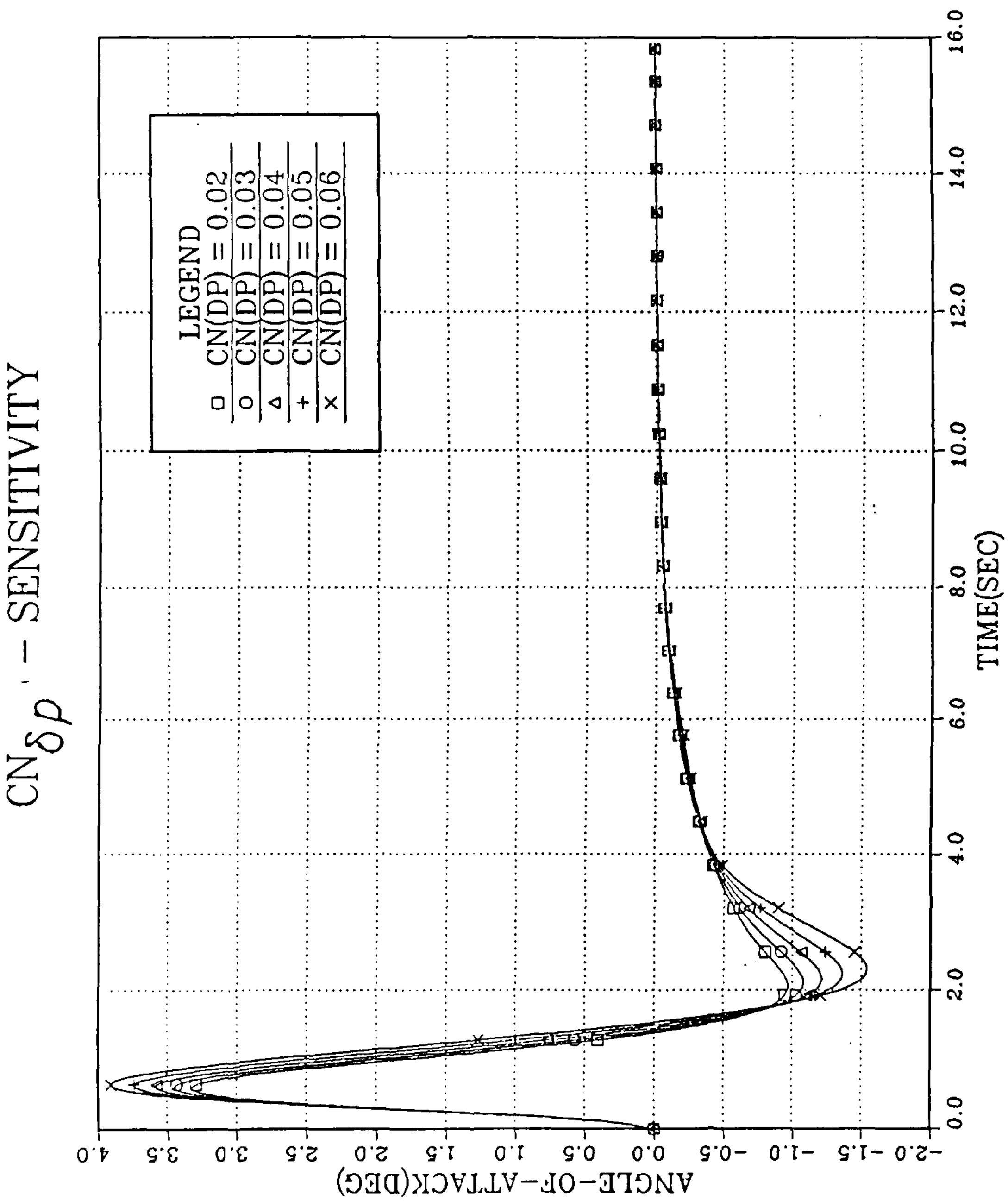

Figure III.13 Angle-of-Attack vs Time for different values of $C_{N\delta p}$

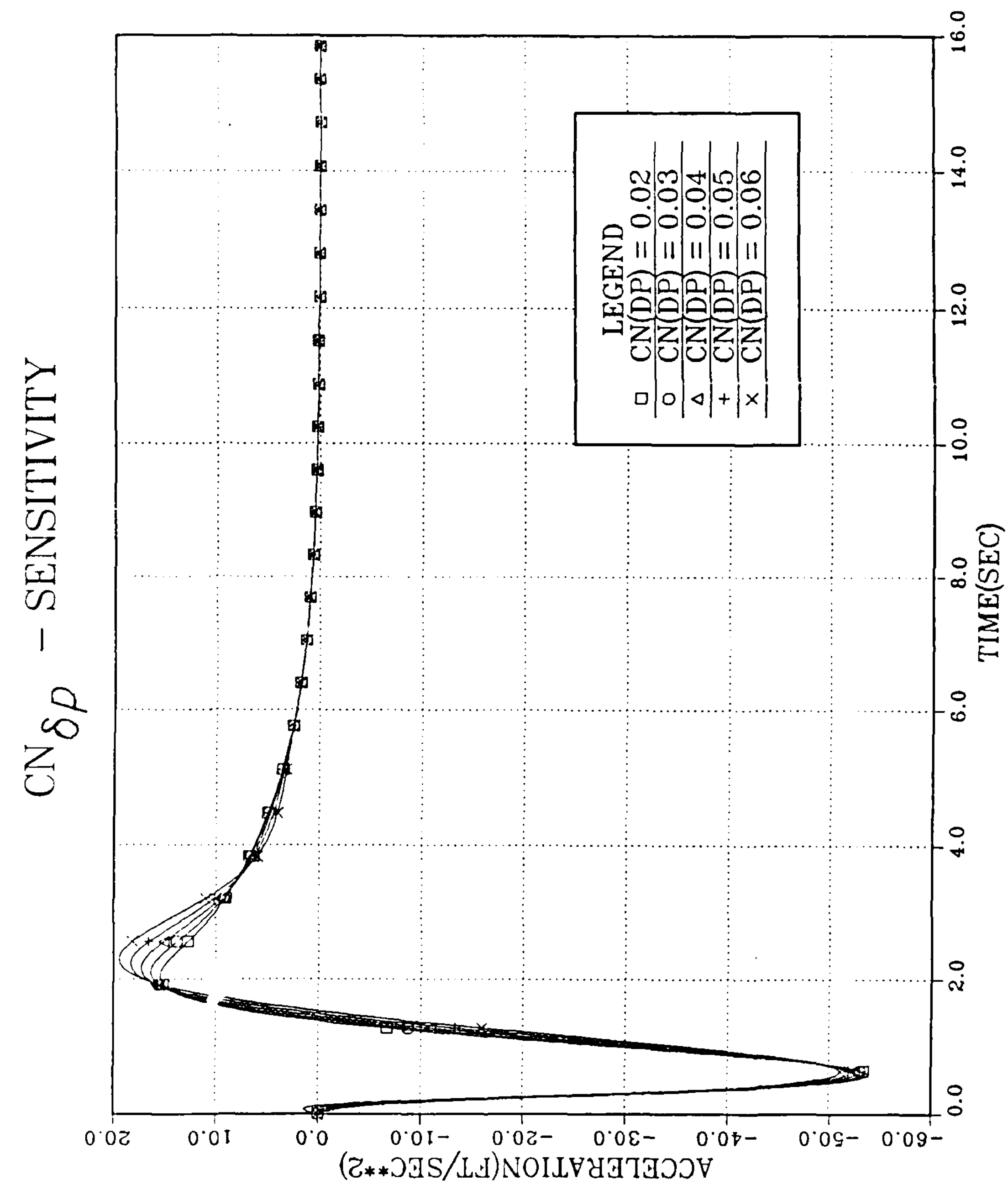

Figure III.14 Acceleration vs Time
for different values of $C_{N\delta p}$

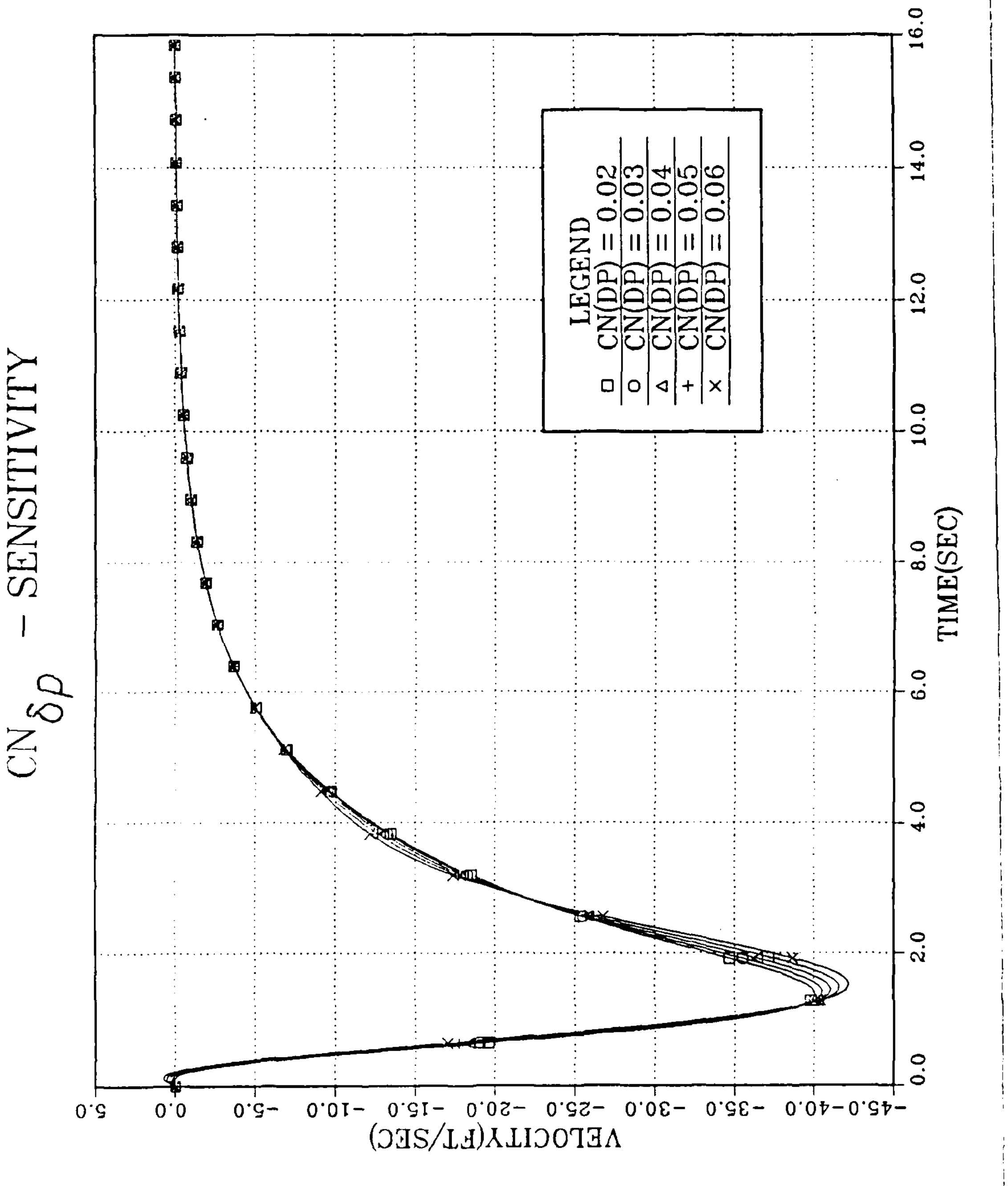

Figure III.15 Velocity vs Time for different values of $C_{N\delta p}$

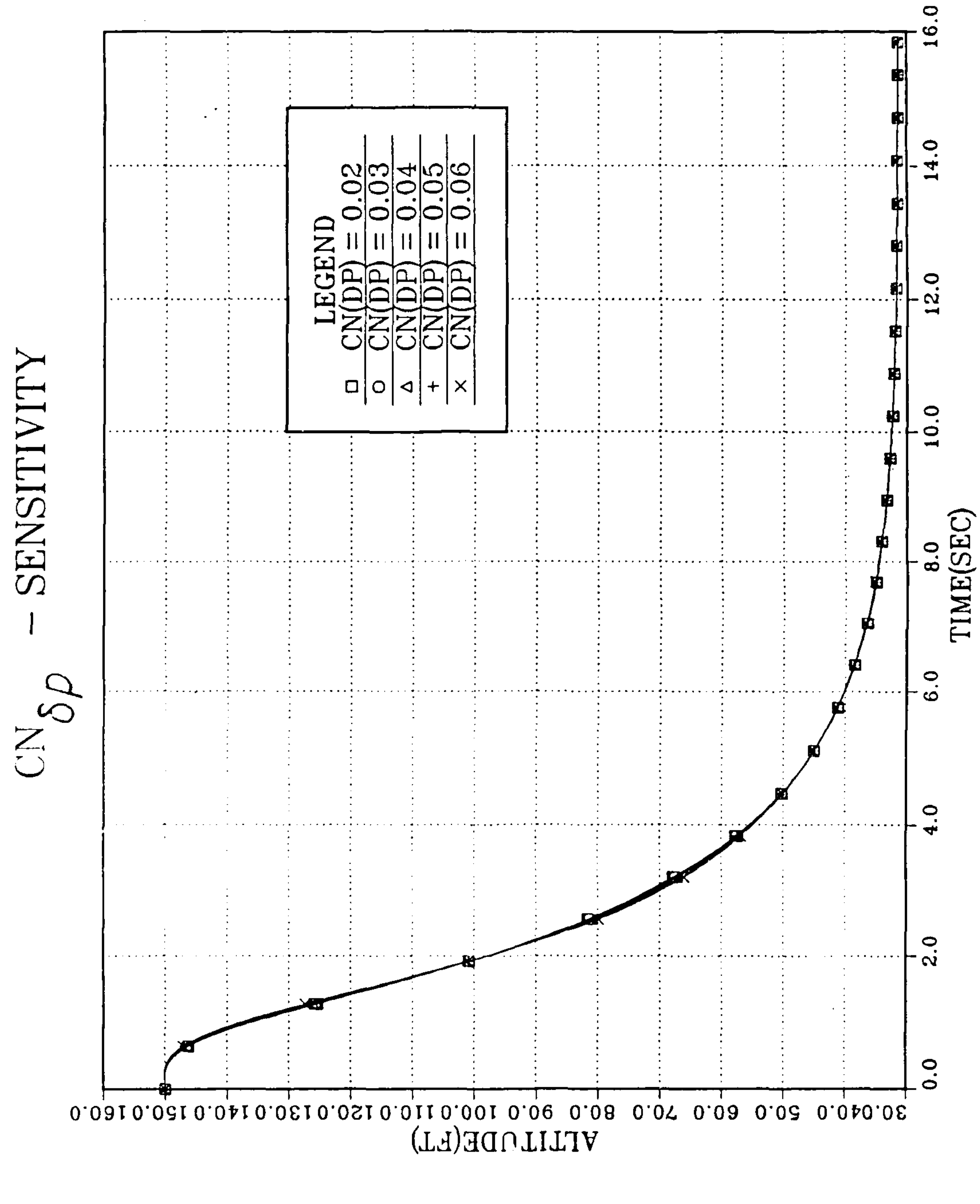

Figure III.16 Altitude vs Time
for different values of $C_{N\delta p}$

IV. ROBUSTNESS ANALYSIS

A. INTRODUCTION

In the previous chapter we take into consideration the change in the eigenvalues and in the time response of the system due to variations on the aerodynamic parameters that take part into the model; under the robustness analysis we verify what we can expect of the system when it is affected by any kind of perturbation.

The main idea is to use the robustness analysis to design a new set of feedback gains in order to have the same behavior in different situations and/or environments which act as perturbations to our missile.

In classical frequency domain, for single-input single-output systems(SISO), a robust design can be achieved using Bode, Nyquist or Nichols plots. With these techniques it is possible to define gains that give us gain and phase margins for a "robust" system. For multi-input multi-output systems (MIMO), the classical techniques are no longer valid.

Taking this in consideration, we have analysed the robustness of the original design by means of the minimum singular value of the return difference matrix for different frequencies and we will use the same technique to improve the control system.

In order to show why we are applying this technique, we start with the Nyquist Criterion for a SISO system and extend the analysis to a MIMO system, using the minimum singular value theory, as explained by Lehtomaki, Sandell and Athans in [Ref.6].

B. THE NYQUIST CRITERION

Let's take in consideration the SISO system represented in Figure IV.1.

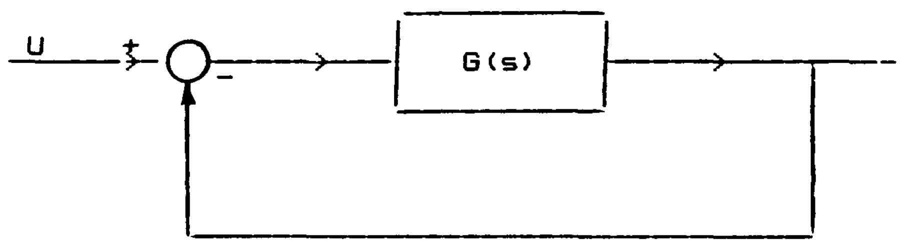

Figure IV.1 Single-Input Single-Output System

where G(s) includes the controller and plant dynamics with unity feedback .

The Nyquist criterion states that if the open loop transfer function G(s) does not have any pole in the right half s plane, then the locus of G(s) will not encircle the point (−1,0) in the Nyquist plot, where jw is substituted for s and the axes represent the real (Re) and imaginary (Im) parts of G(jw) for various frequencies.

A Nyquist Plot is given in Figure IV.2 and it can be used to design feedback gains that ensure a robust system.

The measure of the system stability are Gain and Phase margin obtained from the Figure, as indicated.

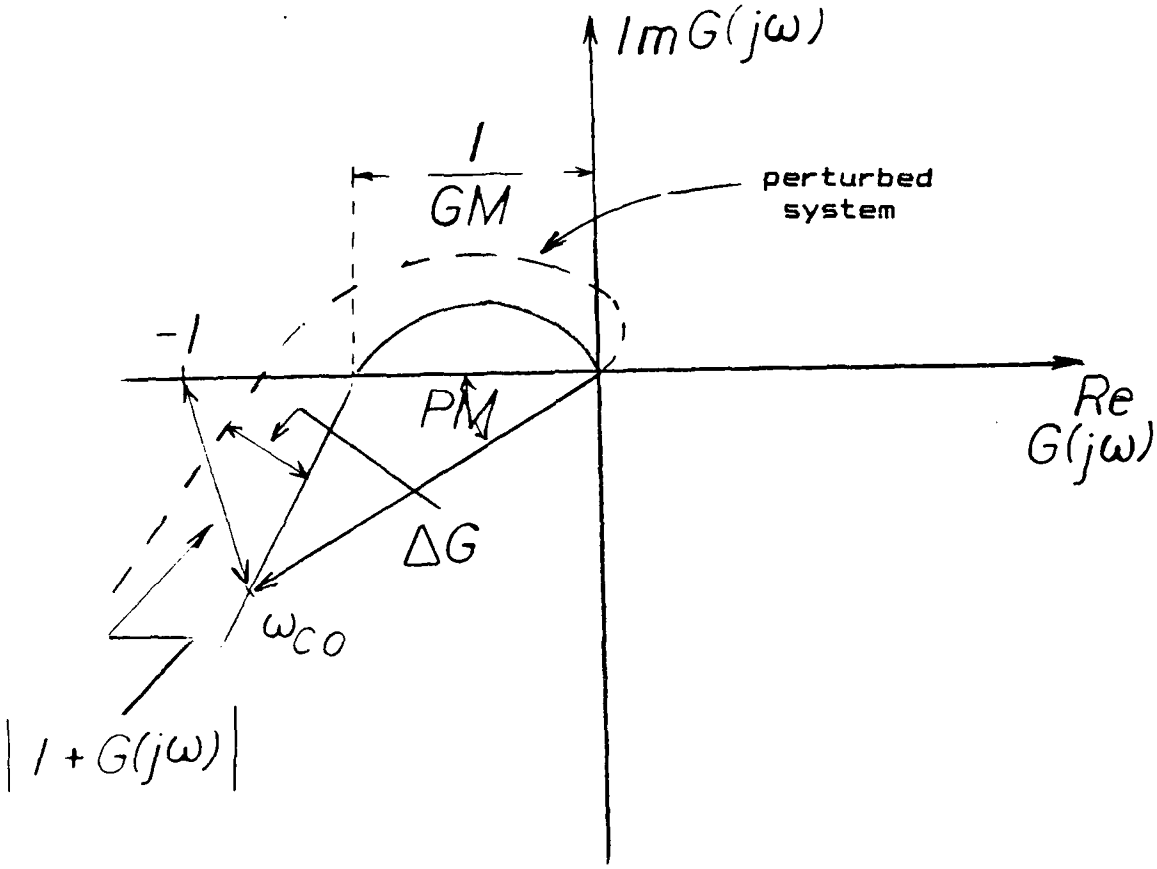

Figure IV.2 Nyquist plot - Stable System

The Phase Margin (PM) is the additional phase lag at the gain crossover frequency required to bring the system to the limit of instability and the Gain Margin (GM) is the reciprocal of the magnitude |G(jw)| at the frequency where the phase angle is -180^{0}. The gain crossover frequency is the frequency at which $|G(jw)|$ is unity.

The Phase and Gain margins are a measure of how close a polar plot is to the $-1 + j0$ point.

Considering the system subject to additive perturbation, as we have in the diagram of Figure IV.3.

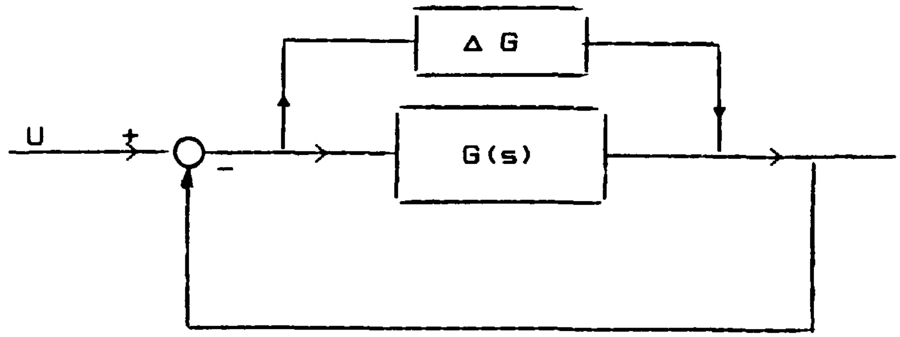

Figure IV.3 Additive Perturbation

In order to have the system stable, despite the perturbation, we need

$$|\Delta G(jw)| < |1 + G(jw)| \qquad \text{(IV.1)}$$

As we can see in Figure IV.2, this condition ensures a stable system.

This idea can be extended to MIMO problems through the use of matrix norms and of applying the Multivariable Nyquist Theorem.

C. MULTI-INPUT MULTI-OUTPUT SYSTEM

The generalization of the SISO theory discussed in the previous section has been made for the MIMO problem.

Let's consider our system as represented in chapter III and apply the transformation in order to have the open loop transfer function between input and output.

$$\dot{x} = A\,x + B\,u \qquad \text{(IV.2)}$$

Applying Laplace Transform to the equation (IV.2), we have

$$sX = AX + BU \qquad \text{(IV.3)}$$

where X and U represent the Laplace Transform of $\frac{dx}{dt}$ and u(t), respectively. Taking the value of X in equation (IV.1)

$$X = [sI - A]^{-1}B\,U \qquad \text{(IV.4)}$$

As

$$Y = C\,X \qquad \text{(IV.5)}$$

We have

$$\frac{Y}{U} = C\,[sI - A]^{-1}B \qquad \text{(IV.4)}$$

This transfer function corresponds to G(s) in Figure IV.4.

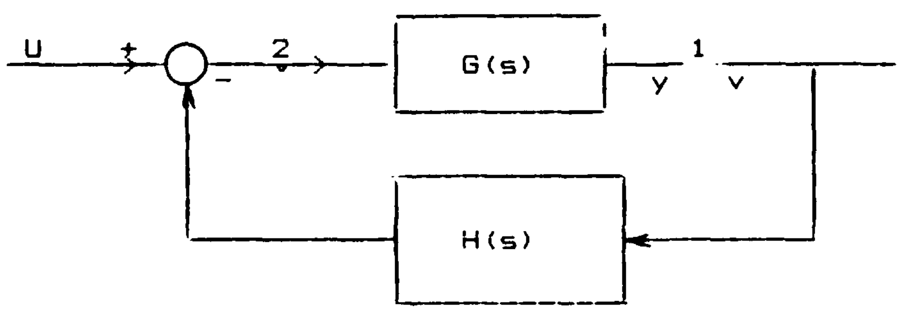

Figure IV.4 Return Difference

From Figure IV.4 it is possible to define the "Return Difference Matrix" (Kwakernaak and Sivan, [Ref.6]) that corresponds to opening the loop at a given point and connecting an external input variable at this point; for example, at point 1. Assuming the input u as zero, we have:

$$Y_1(s) = - G(s)\ H(s)\ V\ (s) \qquad \text{(IV.5)}$$

where V (s) corresponds to the new input variable and $Y_1(s)$ the "returned variable".

The difference between $Y_1(s)$ and V(s) is

$$V(s) - Y_1(s) = [\ I + G(s)H(s)\ ]\ V(s) \qquad \text{(IV.6)}$$

The matrix I + G(s)H(s) is defined as the "Returned Difference Matrix", with G(s) equal to

$$G(s) = C[sI - A]^{-1}B \qquad \text{(IV.7)}$$

As explained by Lehtomaki,Sandell and Athans in [Ref.7], the Multivariable Nyquist Theorem is derived from the relationship

$$\det[I + G(s)H(S)] = \frac{\Phi cl(s)}{\Phi ol(s)} \qquad \text{(IV.9)}$$

where $\Phi cl(s)$ = det(sI − A) corresponds to the characteristic polynomial of the open loop transfer function and $\Phi ol(s)$ = det(sI − A + BF) corresponds to the characteristic polynomial of the closed loop transfer function from the system represented in Figure IV.4.

The multivariable Nyquist theorem which requires that a closed loop stable system have the same number of counterclockwise encirclements of the origin by the locus of the determinant of I + G(s)H(s) as the number of open loop poles that are unstable.

If I + G(s)H(s) is quasi-singular, a small change in G may make the matrix singular ; this causes the det[I + G(s)H(s)] to become zero and the Nyquist encirclement count to change indicating an unstable system.

Basically,in order to analyse the robustness of the system we have to verify how close the return difference matrix is to being singular as a function of frequency (ω).

The natural measurement of the singularity is the minimum "singular value", since this is the tightest norm.

The singular value of a matrix M is defined as

$$\sigma_i = \{\lambda_i (M^H M)\}^{1/2} \qquad \text{(IV.10)}$$

M^H represents the conjugate transpose matrix of a generic M matrix and λ_i any eigenvalue of the product of M^H times M.

If the minimum singular value is close to zero the matrix is quasi-singular and the system is not robust.

If one assumes

$$\underline{\sigma}(I + G(jw)H(jw)) > \alpha_0 \qquad \text{(IV.11)}$$

then it can be shown [Ref. 7] that the gain and phase margins for the system may be represented by equations (IV.12) and (IV.13), i. e.,

Gain margin: $$GM = \frac{1}{1 + \alpha_0} \qquad \text{(IV.12)}$$

Phase margin: $$PM = \pm \cos^{-1}[1 - \frac{\alpha_0^2}{2}] \qquad \text{(IV.13)}$$

Universal gain and phase margins curves, presented by Gordon [Ref. 8] and repeated in Figure IV.5 allow the designer to pick a singular value that corresponds to a desired stability margin for the system to be designed.

From singular value plots the we will be able to identify the critical frequencies or range of frequencies where the singular value is below the chosen level from the Figure IV.5.

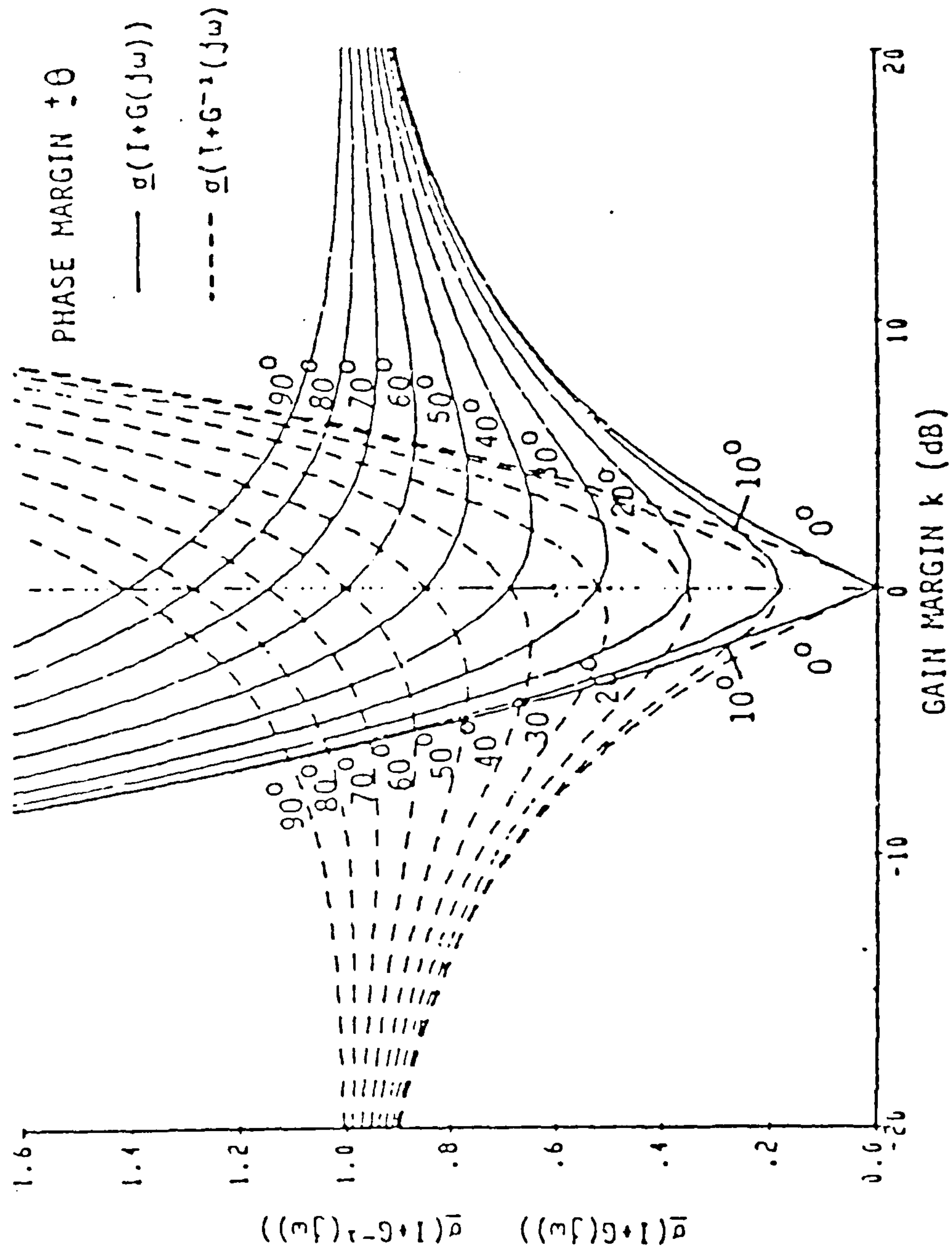

Figure IV.5 Universal Gain and Phase Singular Value Plot

D. SINGULAR VALUE ANALYSIS

Using the program developed by Gordon [Ref.8], with small changes to make it compatible with the size of our system and to input the matrices more easily, we calculate the minimum singular value of the return difference matrix, with the frequency varying from 0 to 50 rad/sec.

The program utilizes the IMSL subroutines to generate the return difference matrix as well as to made the singular value decomposition, given the minimum singular value for different frequencies.

The return difference matrix depends on the point where we open the loop represented in Figure IV.5. For the explained case, considering point 1, we have the "output" difference matrix, that corresponds to I + G(s)H(s); if the loop is open at point 2 we have the "input" case and the return difference matrix will be I + H(s)G(s).

From the Figure IV.5 we choose as reference, for our analysis, the minimum singular value as 0.6 that gives us a phase margin of about 35^0 to characterize a "robust" system.

The minimum input singular value is plotted in Figure IV.6.

It shows that the system is robust, except for frequencies between 0.6 and 5.0 rad/sec.

The results related to the minimum output singular value are in Figure IV.7 and the robustness of the system is poor from 0 to 18 rad/sec.

Those results indicate that the behavior of the system under perturbations should present problem for frequencies below 10 rad/sec.

To confirm this expectation, we have plotted the BODE diagram (magnitude) for the closed loop system considering the angle-of-attack and altitude with respect to both inputs (acceleration and desired altitude).

The diagrams are on Figures IV.8 to IV.11, where can see the attenuation at the low frequencies indicating that we can not expect good response at those frequencies and one could further expect problems with robustness as is evident from the singular value plots.

On the next chapter we try to .mprove the system designing a new set of feedback gains, in order to have a robust system.

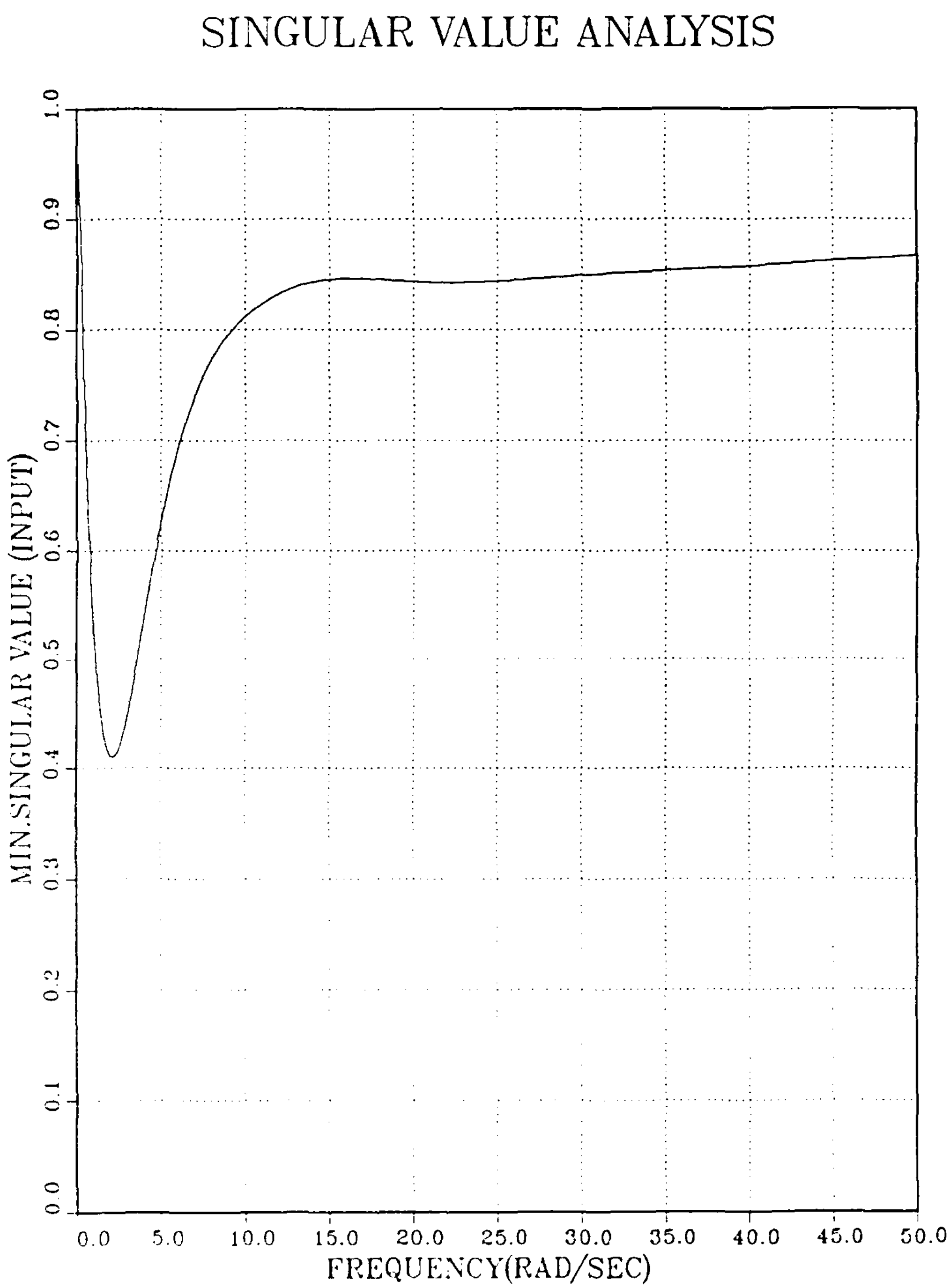

Figure IV.6 Input Minimum Singular Value

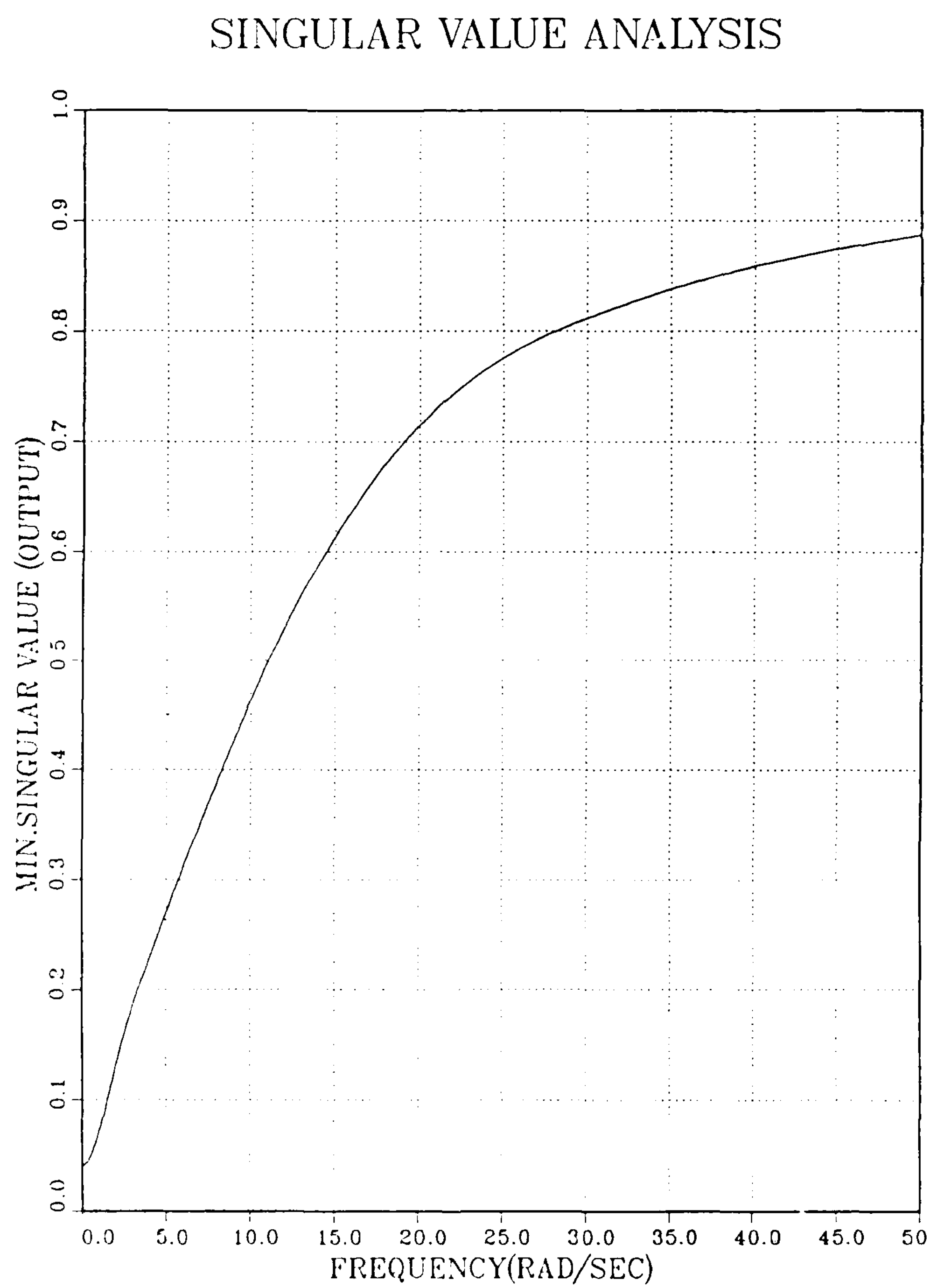

Figure IV.7 Output Minimum Singular Value

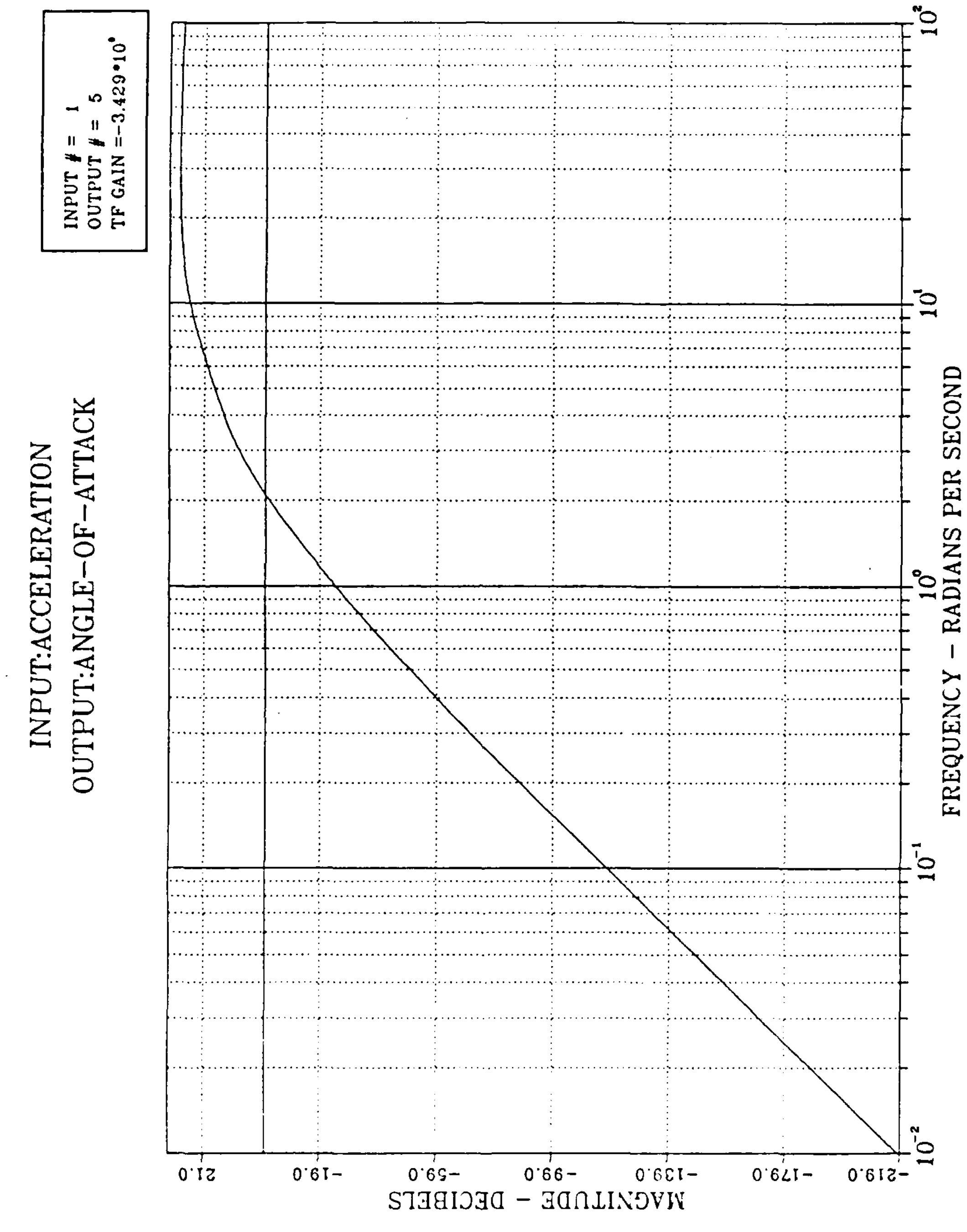

Figure IV.8 Bode Magnitude - Angle-of-Attack vs. Acceleration

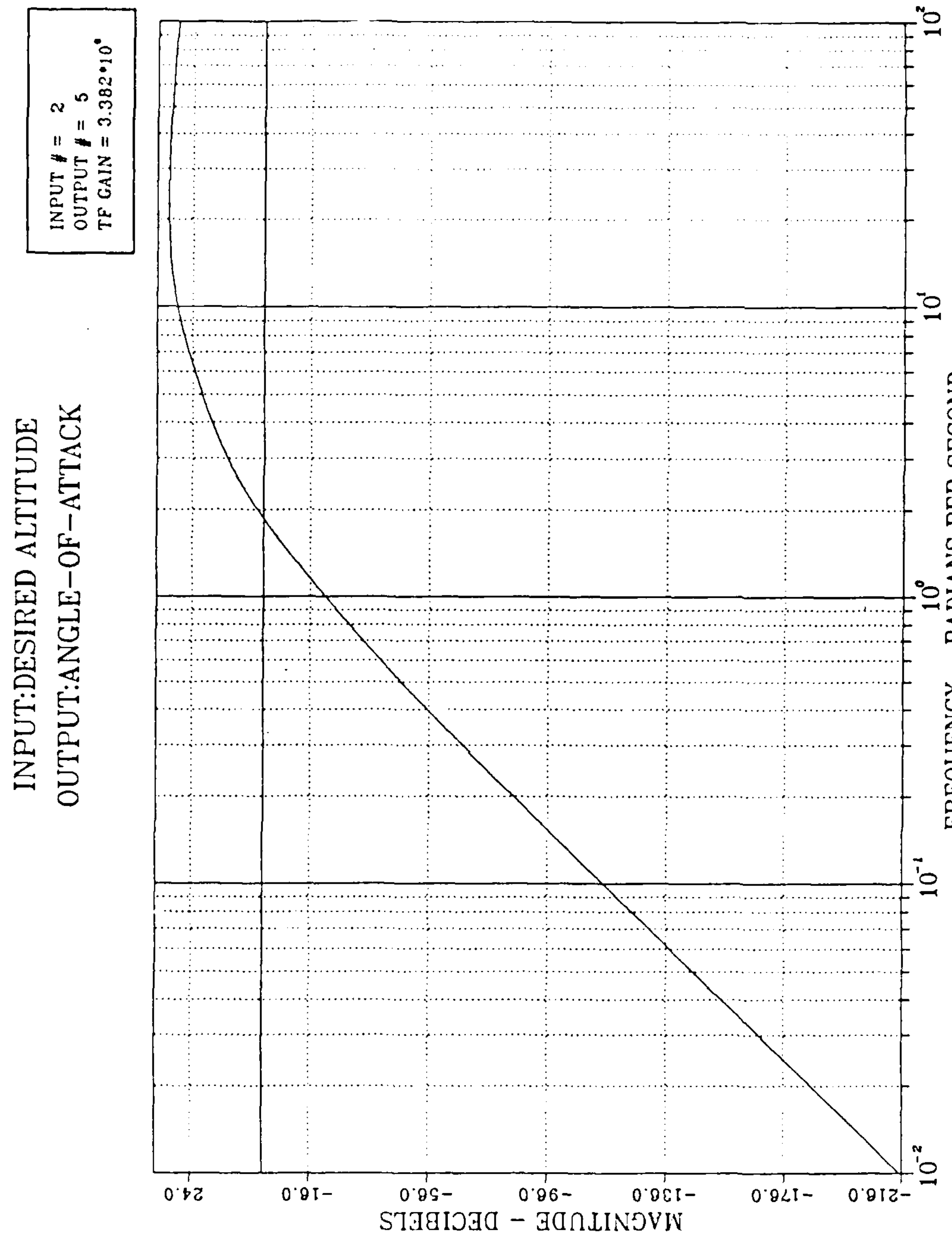

Figure IV.9 Bode Magnitude - Angle-of-Attack vs. Desired Altitude

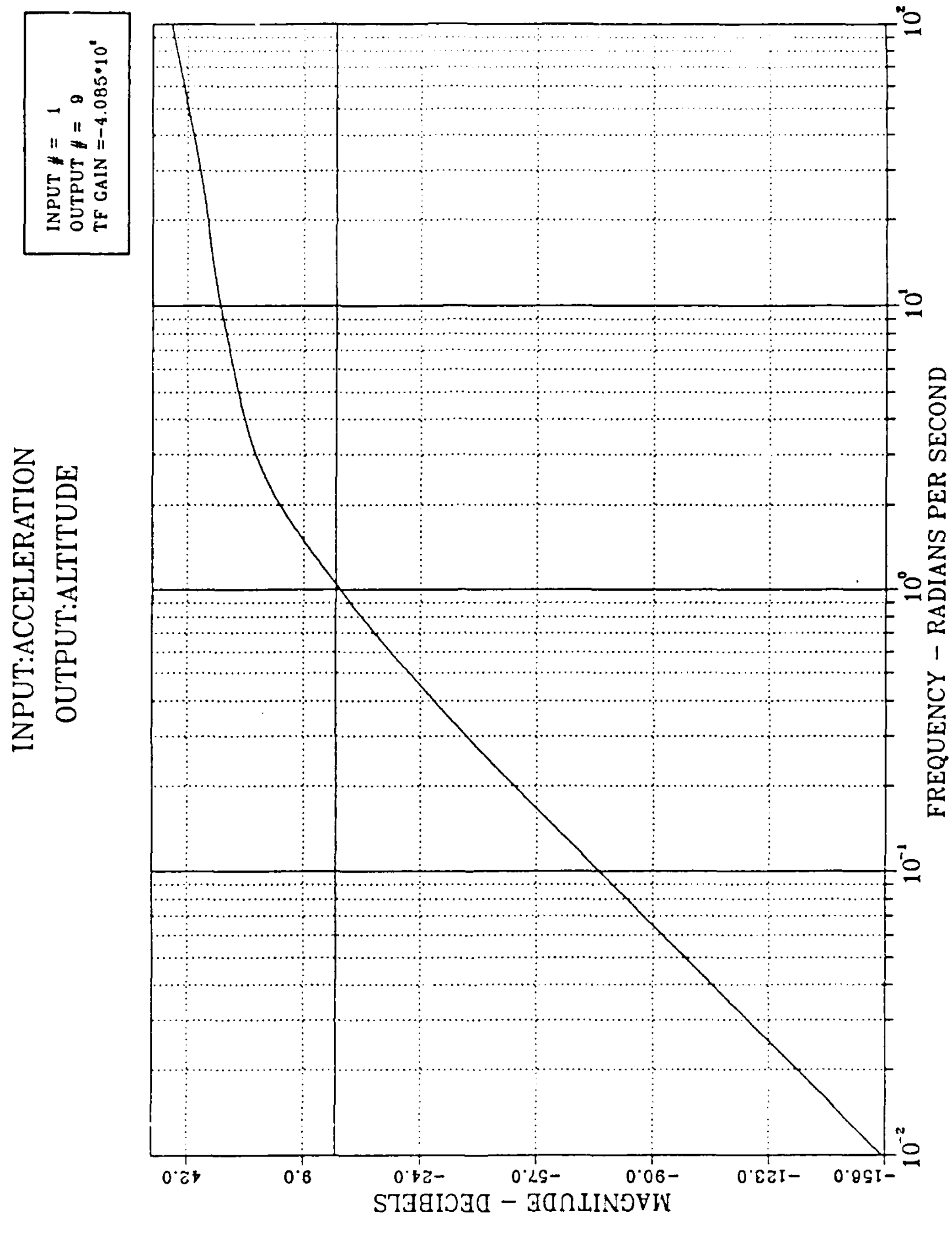

Figure IV.10 Bode Magnitude - Altitude vs. Acceleration

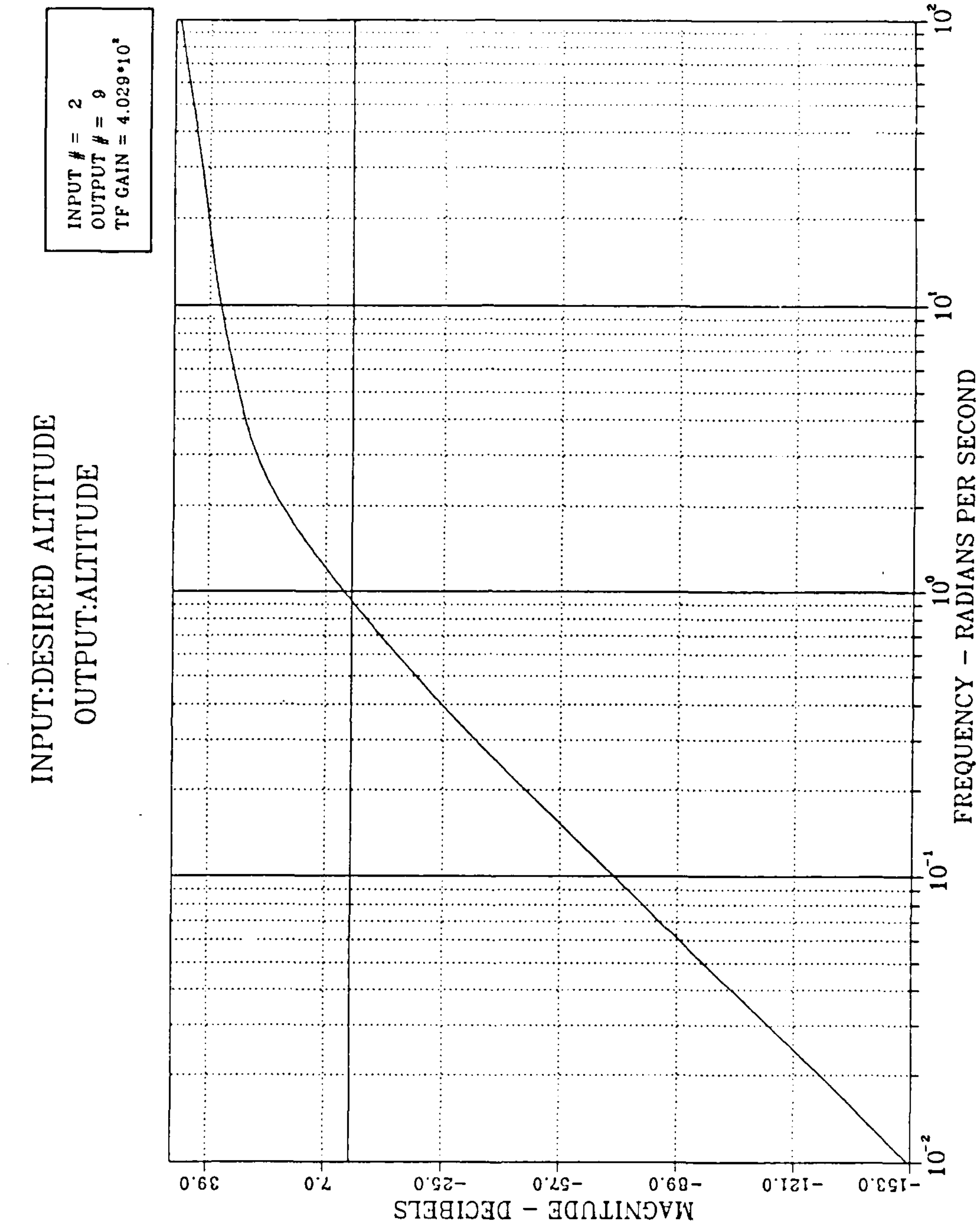

Figure IV.11 Bode Magnitude - Altitude vs. Desired Altitude

V. IMPROVING THE DESIGN

Under this chapter we try to improve the system by calculating new feedback gains that will yield a more robust system, taking into consideration the results indicated in Figures IV.6 and IV.7.

The POP'AR program, developed by Gordon [Ref.8] will be used. A numerical optimization technique is applied in order to increase the minimum singular value of the considered return difference matrix, therefore, a resultant robustness of the design.

The development in this chapter will be preceded by comments on numerical optimization and a description of the computer program.

A. OPTIMIZATION

The optimization was accomplished by means of the Automated Design Synthesis Program (ADS) developed by Vanderplaats [Ref.9].

The purpose of ADS as of others numerical optimization routines is to find the "best" possible solution for the problem, starting from an initial set of variables and updating the design iteractively. The problems can develop in convergence of the method and in the computer time needed. If the problem has multiple solutions, the optimization does not always lead to the absolute optimum.

The ADS program is designed as a black box optimizer which allows the user to choose combinations of one dimensional search, optimization algorithm and strategy.

ADS is used as a subroutine and the parameters that correspond to the different applications are chosen by the user, as explained in [Ref.9].

The preferred method used in the ADS applications is referred to as Sequential Unconstrained Minimization Techniques and can be considered as a method that starts with an objective function and the constrains combined into an augmented objective function and then minimizing this function as if the problem was unconstrained.

ADS employs penalty function techniques as well as an Augmented Lagrange Multiplier.

As presented in [Ref.8], the iteration between the user's program and the ADS routine can be represented by the block diagram of Figure V.1; where the "info" parameter is used as a "flag" for a dialogue between the user and the ADS routines.

The details for utilization of ADS are presented by Vanderplats in [Ref.8].

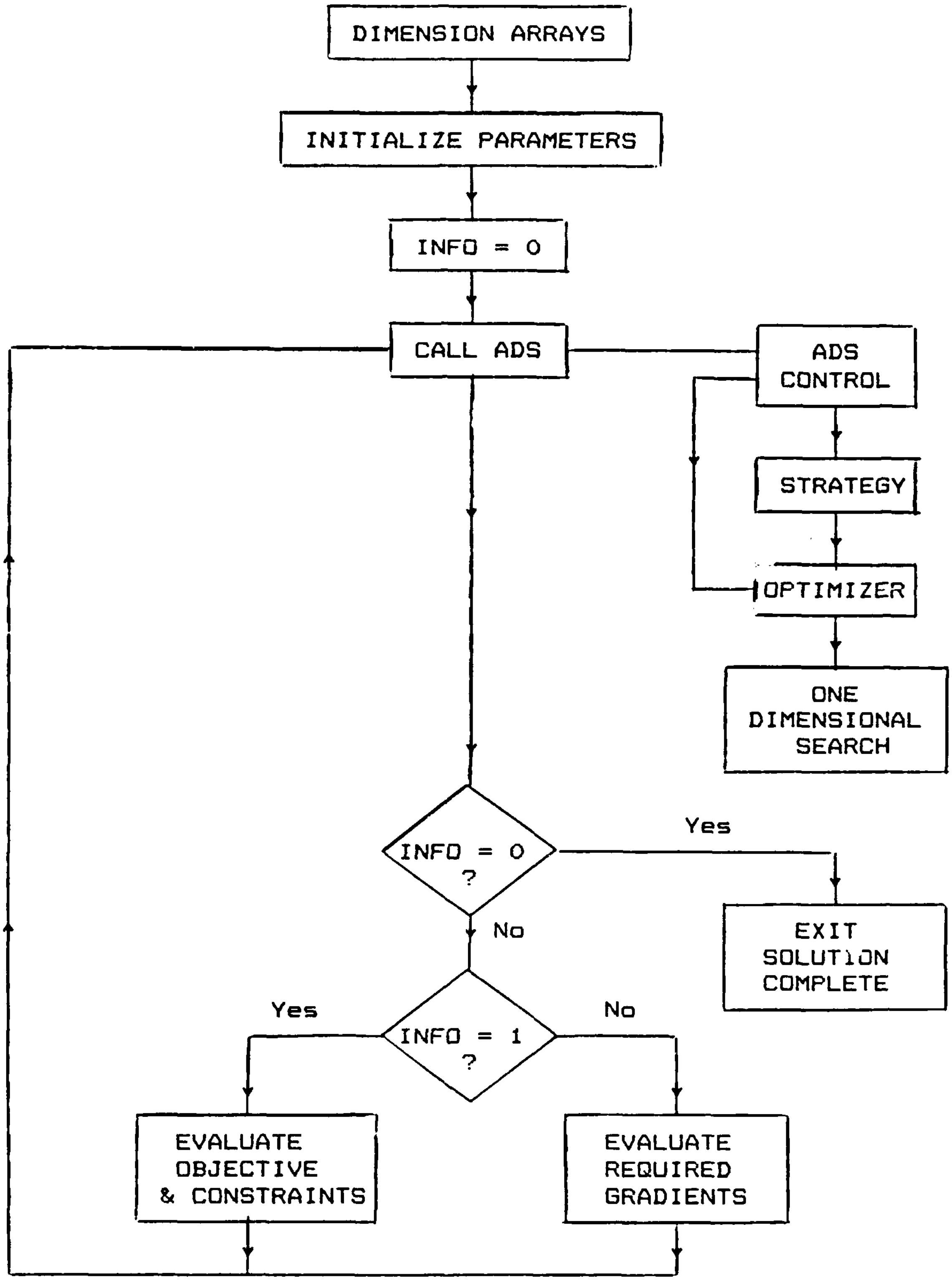

Figure V.1 Organization of ADS Program

B. POPLAR PROGRAM

The Gordon's program has the capability of calculating the feedback gains that improve the response of the system either by pole placement and/or by increasing the minimum singular value of the return difference matrix.

The main objective is to improve the robustness by increasing the minimum singular value above a desired level chosen from the universal curve reproduced in Figure IV.5.

"Optimum" values of the feedback are calculated by minimization of an Objective function whose pole placement part is

$$OBJ = \sum_{i=1}^{l} (\lambda_{R_{di}} - \lambda_{R_i})^2 + (\lambda_{I_{di}} - \lambda_{I_i})^2$$

where

$\lambda_{R_{di}}$ - real part of the desired eigenvalue i;

λ_{R_i} - real part of the computed eigenvalue;

$\lambda_{I_{di}}$ - imaginary part of the desired eigenvalue i;

λ_{I_i} - imaginary part of the computed eigenvalue i;

and, for the minimum singular value optimization, we have

$$OBJ = \sum_{j} \{ \max [0 , \sigma_d - \underline{\sigma} (jw,p)] \}^2$$

where σ_d indicates the desired minimum singular value and $\underline{\sigma}$ is the minimum singular value at a certain frequency.

The optimization procedure changes the feedback gains until the minimum singular value is raised to the desired level.

The pole placement and robustness program calculates the return difference matrix to be considered, working in the complex space (as the analysis is conducted in the frequency domain) and the objective functions, calling the ADS routine to make the optimization.

Input's for the program are the matrices of the state representation of our missile (A , B , C and F),where the observation matrix (C) was specified as Identity because we are assuming all the states are observable as well as controllable.

Other inputs are the desired minimum singular value, desired pole locations and frequency interval to be considered.

In our application of POPLAR the initial values of the feedback gains are those of the original system and the parameters for application of the ADS program are:

Strategy - Augmented Lagrange multiplier;

Optimizer - BFGS variable metric method for unconstrained minimization;

One-Dimensional Search - Polynomial interpolation.

Two situations will be considered; the INPUT - minimum singular value, where the return difference matrix is I + HG and OUTPUT-minimum singular value with I + GH as the return difference matrix.

C. INPUT - MINIMUM SINGULAR VALUE

The program was first run to improve the minimum singular value for the input situation within the same interval of frequencies used in Chapter IV.

Due to the large CPU time involved, the program was run for the critical band of frequencies and the result evaluated for the complete interval.

The minimum singular value taken as reference from the Universal curve was 0.6 that implies a phase margin of 35^0.

We have an objective function taking into consideration the improvement of the minimum singular value as well as a part corresponding to the pole placement. The main goal is to improve the singular value but with a solution such that the poles are in positions where the time response satisfies the requirements.

The weight of the pole placement part was considered as 10% of the singular value part.

The program was started considering only part of the feedback matrix as free parameters, increasing this number until a reasonable solution was found.

The best solution was found keeping the original feedback gains and calculating gains to feedback all the other states to generate the commanded acceleration.

The computer output of the best solution with the corresponding inputs is given in Appendix B.

The feedback gains that improve the robustness of the system are the following

$$F = \begin{bmatrix} 0.04 & -1.15 & 0.03 & -1.7 & -9.07 & 0.32 & 7.07 & 0.049 & 0.986 & 1.4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The plot of the minimum singular value for different frequencies is given in the Figure V.2, along with the values for the original system.

As we can see the minimum singular value was increased in the critical values of frequency with the minimum changing from 0.40 to 0.63 that indicates an improvement in the phase margin from 20^0 to 35^0. Also the bandwidth where the system was less robust was changed from 5.0 rad/sec to approximately 1.0 rad/sec.

The robustness of the control system for all the considered frequencies is better, arriving close to 0.9 with a considerable improvement at low frequencies.

Using the software Controls, the time response was plotted in Figure V.3.

The maximum and minimum values of the angle-of-attack are smaller than those of the original system; the maximum acceleration and velocity are slightly lower and an undershoot of about 10% appears on the altitude, that could be a problem in some flight conditions.

We now turn to a consideration of the output return difference matrix although this is less important from a practical viewpoint.

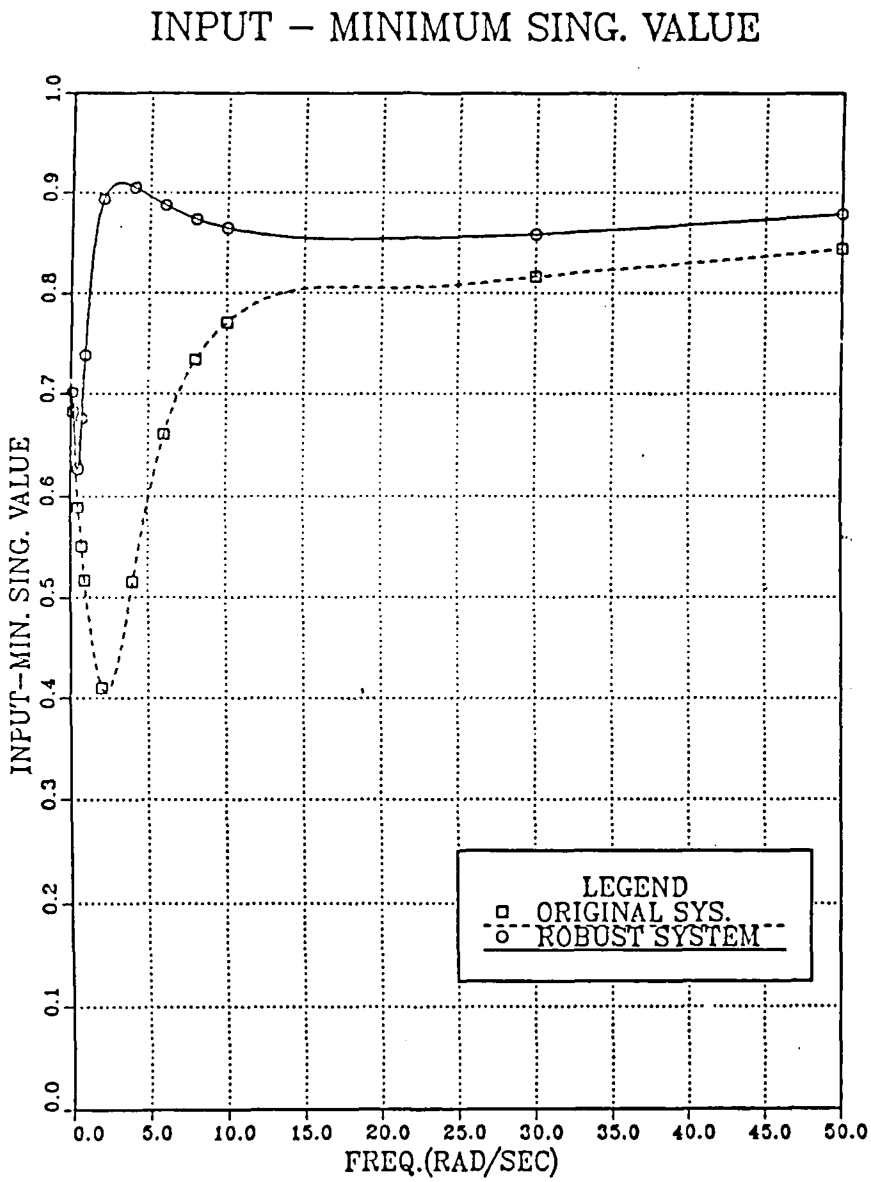

Figure V.2 - Improvement in the INPUT-Minimum Singular value

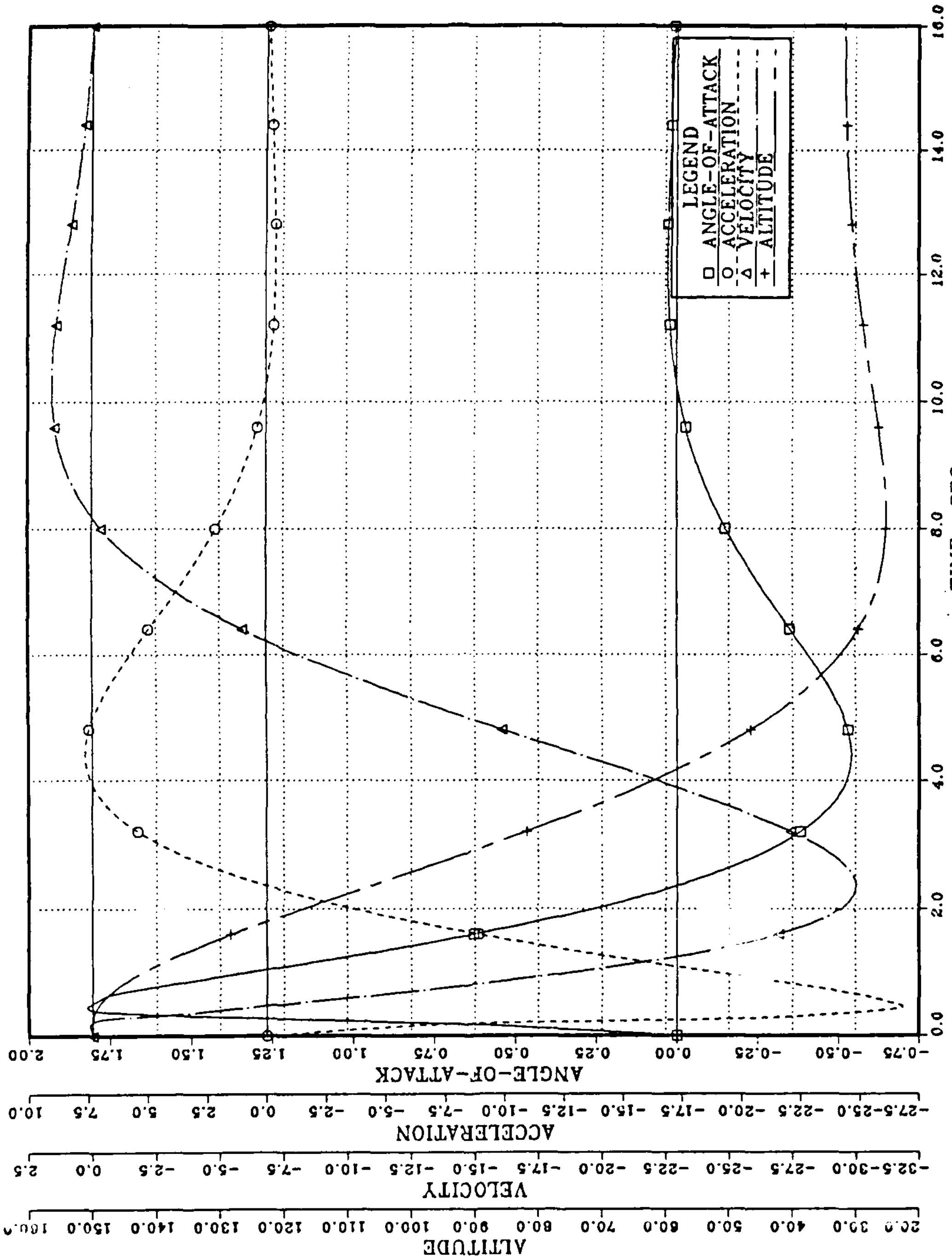

Figure V.3 - Time Response for the Improved System (INPUT)

D. OUTPUT - MINIMUM SINGULAR VALUE

In the Figure V.4 we have plotted the output minimum singular values that correspond to the run where the input singular values were improved, as well as the original values.

As a considerable degradation of the output singular values was evident, we try, in the last part of this work, to arrive to a situation where the system could be reasonable robust in both cases, increasing the output singular value even with some reduction at the input situation.

After several runs of the program, using different combinations of optimization techniques with different starting points, it was verified that due to the characteristics of the plant, that increases in the input singular values resulted in decreases in the output situation.

In order to obtain some improvement, an effort was made to modify the system.

The first step was to look at the controllability matrix of our model, represented by

1.855632D-01	4.948168D+02
2.725664D+13	-2.690449D+13
2.725664D+13	-2.690449D+13
2.861121D+00	-2.822009D+00
2.201811D+01	2.171710D+01

-6.781791D+00	6.689111D+00
-2.286975D+00	2.255563D+00
9.855433D+00	-9.719614D+00
-1.508382D+01	1.487685D+01
4.905487D-01	-4.264398D-01

As we can verify, all the elements are non zero, this means that the poles can placed at any desired position; but a close look at the 2nd and 3th rows indicates very high order numbers compared to the others. This situation generates some numerical difficulties in pole placement.

Using engineering judgement, we have to define what kind of changes have to be made for achieving a more robust system.

First we try to get a balanced A-matrix by changing units, i. e., the angles will be in radians instead of degrees and the pitch angular rate in radians per second instead of degrees per second. Despite some reduction on the numbers was not possible to arrive to a satisfactory solution.

More positive effect was obtained by adding the effect of the angle-of-attack in the controller (see Figure II.3).

As the original system has the output singular values higher than 0.5 for frequencies above 10 rad/sec, we have kept the original feedback gains and calculated gains to feedback only the angle-of-attack, pitch rate and commanded actuator.

The gains are the following:

$$F = \begin{bmatrix} 0 & 0 & 0 & 0.037 & 0.027 & 0 & -0.02 & 0.049 & 0.986 & 1.404 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The result is presented in Figure V.5, compared with the original singular values and those from the improvement in the input case.

The output singular values, compared with the original was slightly improved for frequencies below 5.0 rad/sec.

Resultant input singular values are plotted in Figure V.6, and they are lower than those from the improvement in the input situation but a little higher than the original.

The time response were practically unchanged with respect to the original model and it is presented in Figure V.7.

Further analysis is needed to obtain a solution with high output singular values.

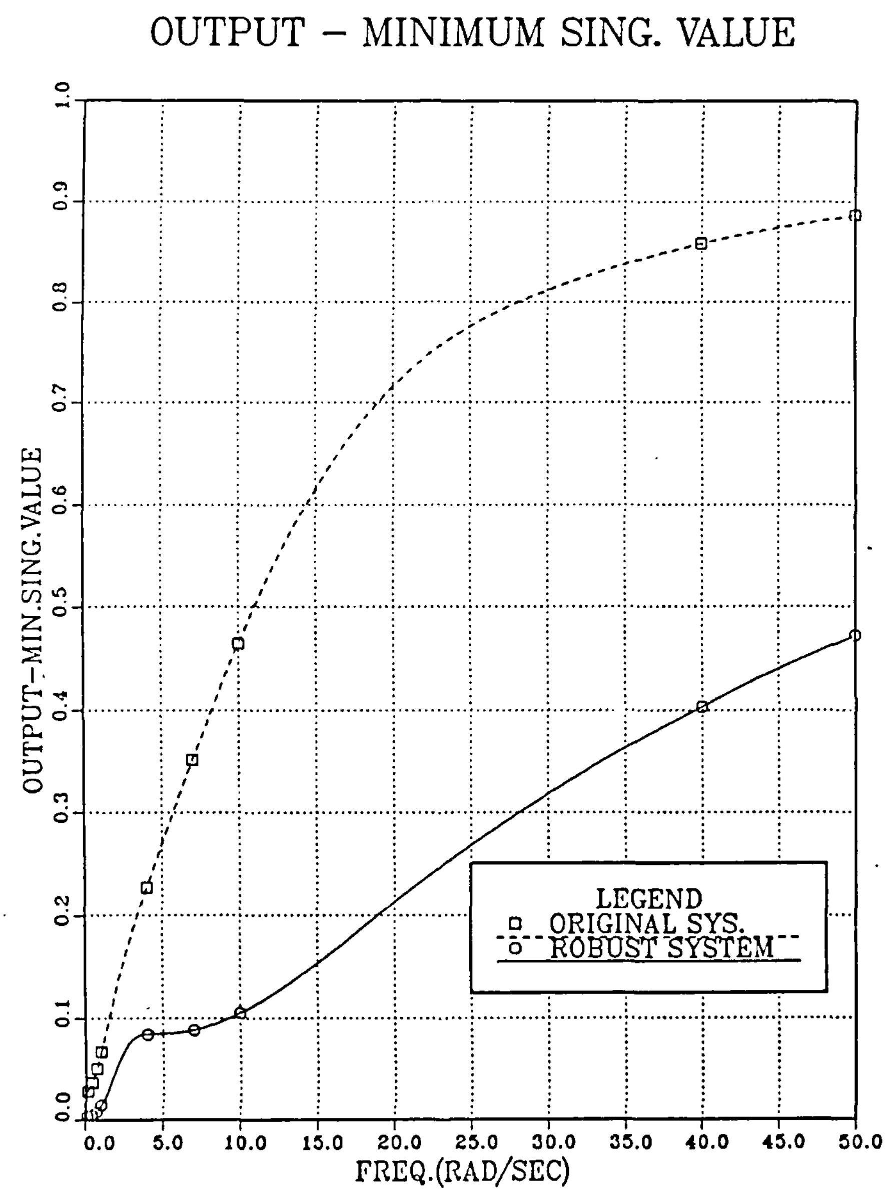

Figure V.4 Output Singular Value Resulted from the Improvement in the Input Singular Value

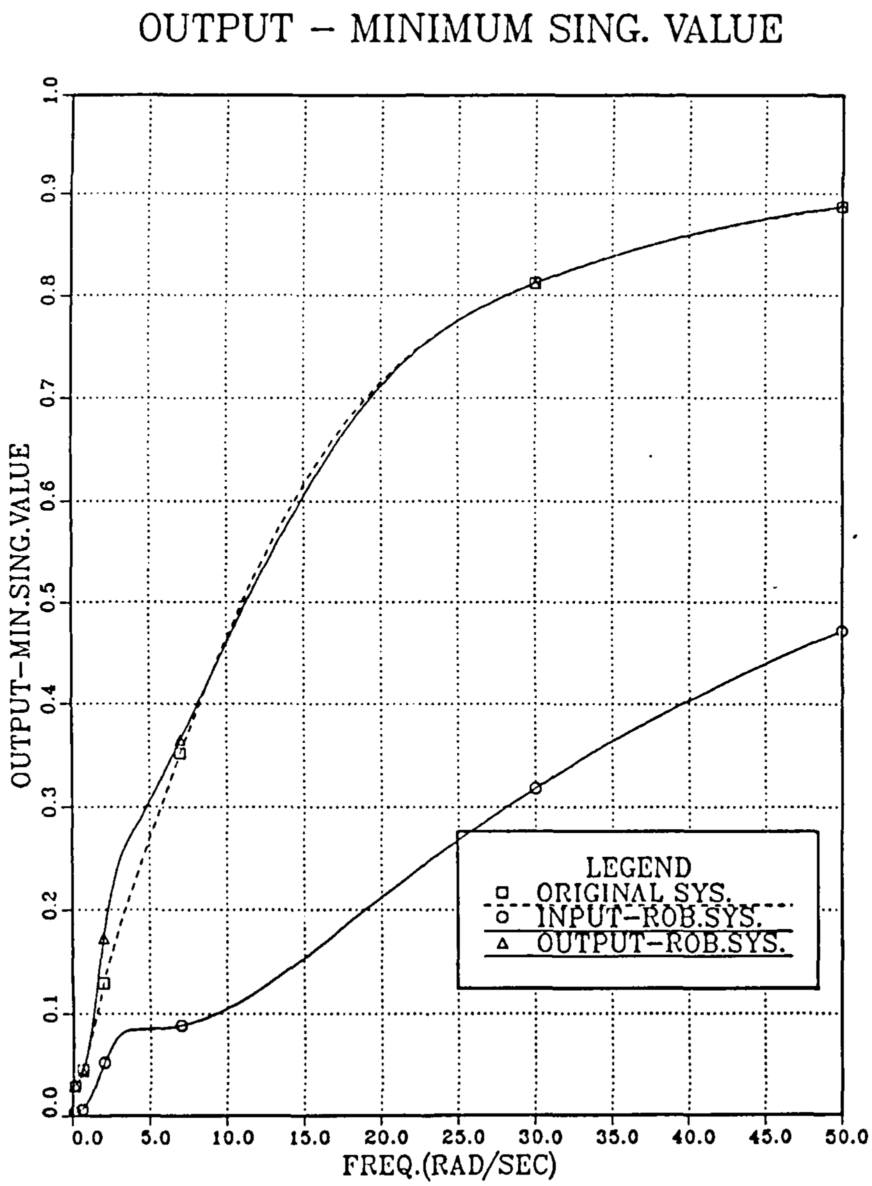

Figure V.5 Output Singular Value

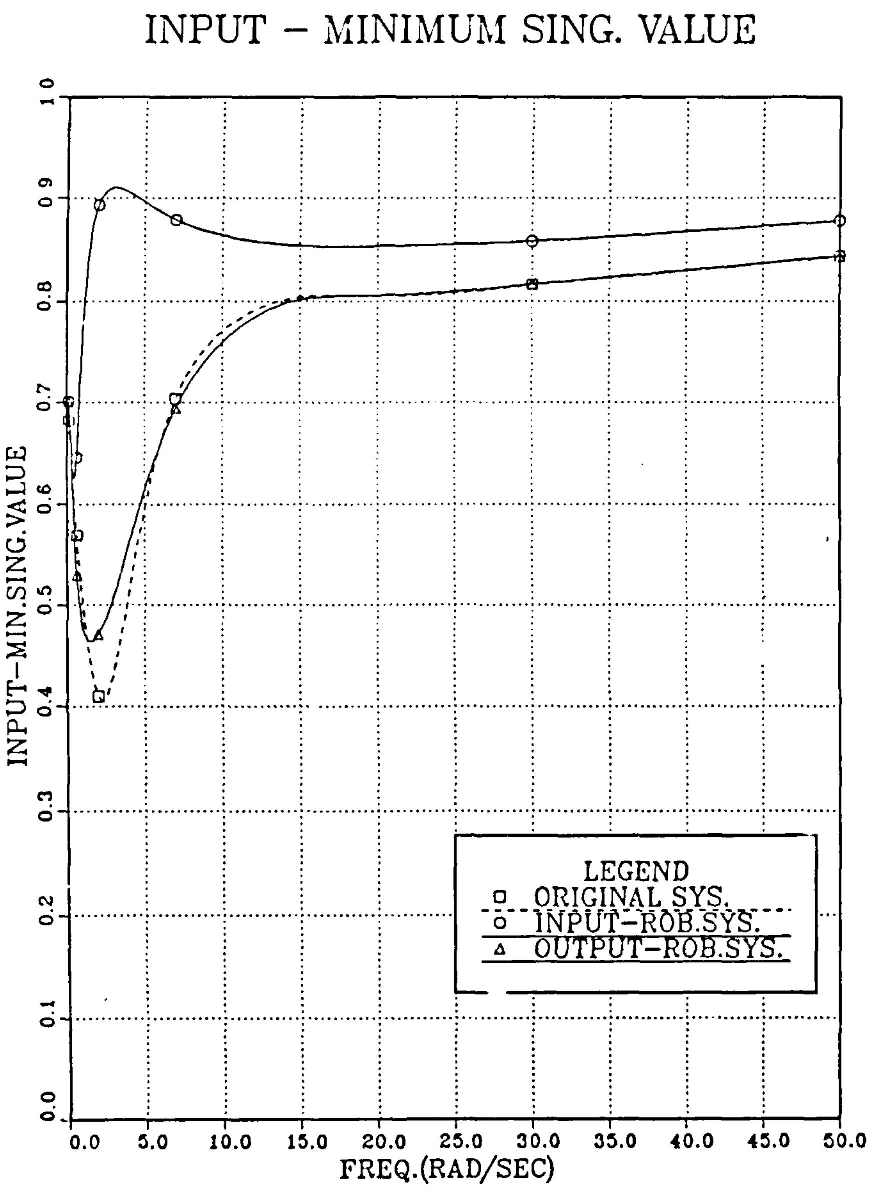

Figure V.6 Final Result in the Input Singular Value

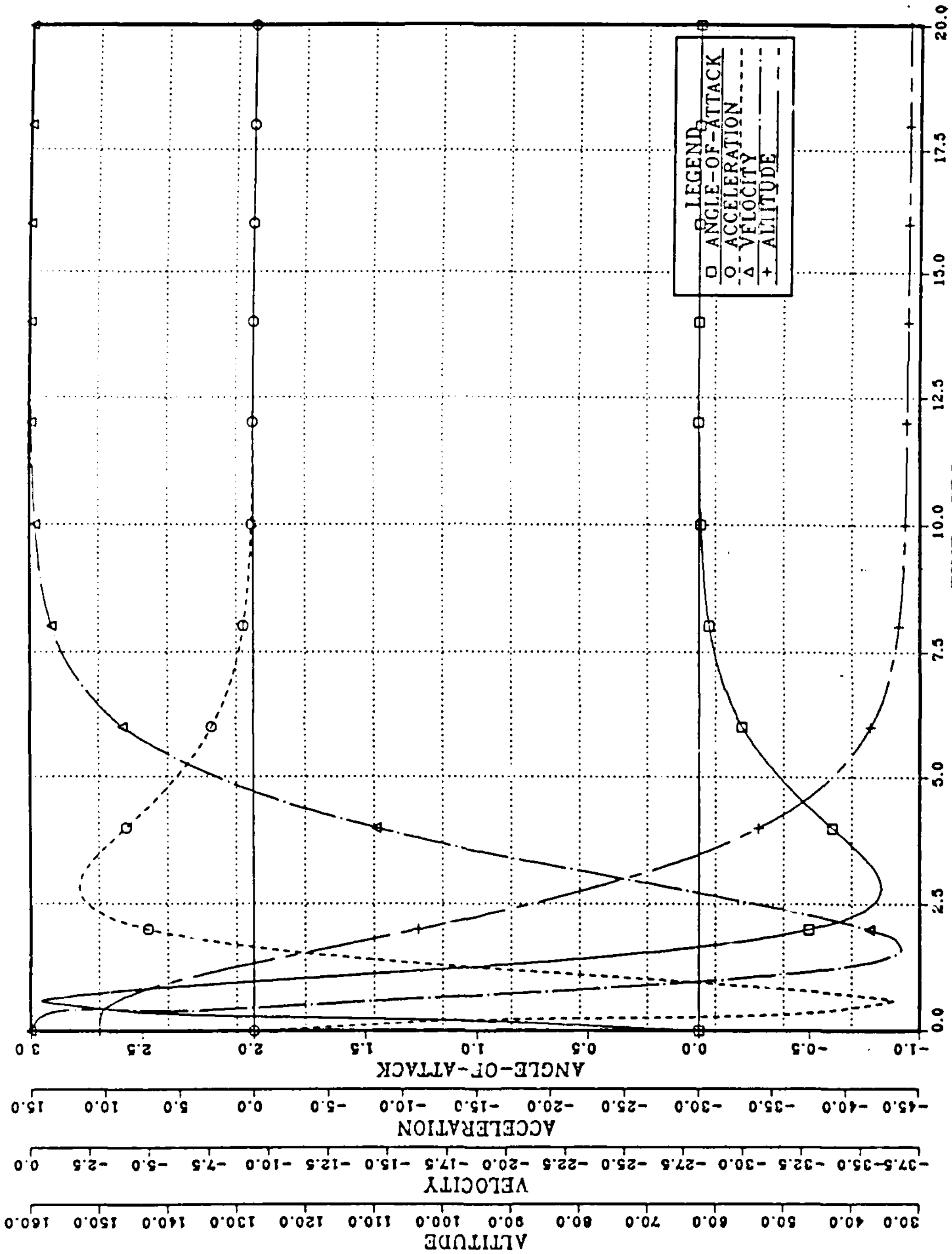

Figure V.7 Final Time Rasponse

VI. CONCLUSIONS

The sensitivity analysis based in the change in eigenvalues represents an important role in the design.

As was demonstrated for the studied missile its behavior corresponds to a system insensitive to the variation of the considered aerodynamic parameters.

The analysis gives a good indication of what parameters have to be precisely determined.

The robustness analysis also demonstrated how useful the singular value analysis is as an auxiliary tool for the designer.

Singular value analysis indicates that the system is robust to input perturbations but is deficient in robustness to output perturbations.

The physical nature of the problem indicates that the prime concern should be with respect to the input perturbations; the output perturbations are of minor concern but should be kept in mind if unusual conditions should be encountered by the missile.

On the MIMO design a commomly used method is the Linear Quadratic analysis where the performance levels are reached by adjusting weighting terms in the "cost" function, but the results of that method, for non-diagonal R matrices, do not necessarily imply a robust system. Using the return difference matrix we can improve the robustness of the design.

One advantage of this procedure is that it permits a high level of interaction between the system and the designer.

Further problems of a computer nature (i.e., large CPU time) are encountered with a high number of states. Analysis must be confined to a small frequency range of low singular values and the complete system return matrix singular value are calculated with the determined feedback gains.

One aspect that should be considered in future development is to improve the cpu time used when handle a system with a high number of states.

APPENDIX A

MISSILE DATA

Under this Appendix the missile sizing, mass properties and aerodynamic parameters are presented as given by Arrow [Ref.1], including the figures.

The missile is 1/6 scale of the actual circular missile configuration and is reproduced in Figure A.1. It is tail controlled using four identical control surfaces located with $\pm 30^0$ dihedral.

A. GEOMETRY AND MASS PROPERTIES

In table VI we have the size and mass properties, with the respactive values, used for development of the state equations.

Only the uncoupled pitch channel was considered, assuming no roll movement.

Table VI

MISSILE GEOMETRY AND MASS PROPERTIES

Weight	W	2525 lbs
Length	l	168 in
Diameter	d	24 in
Reference Area for Coeff.	S	π ft^2
Moment of Inertia about y	I_{yy}	804 slug-ft^2

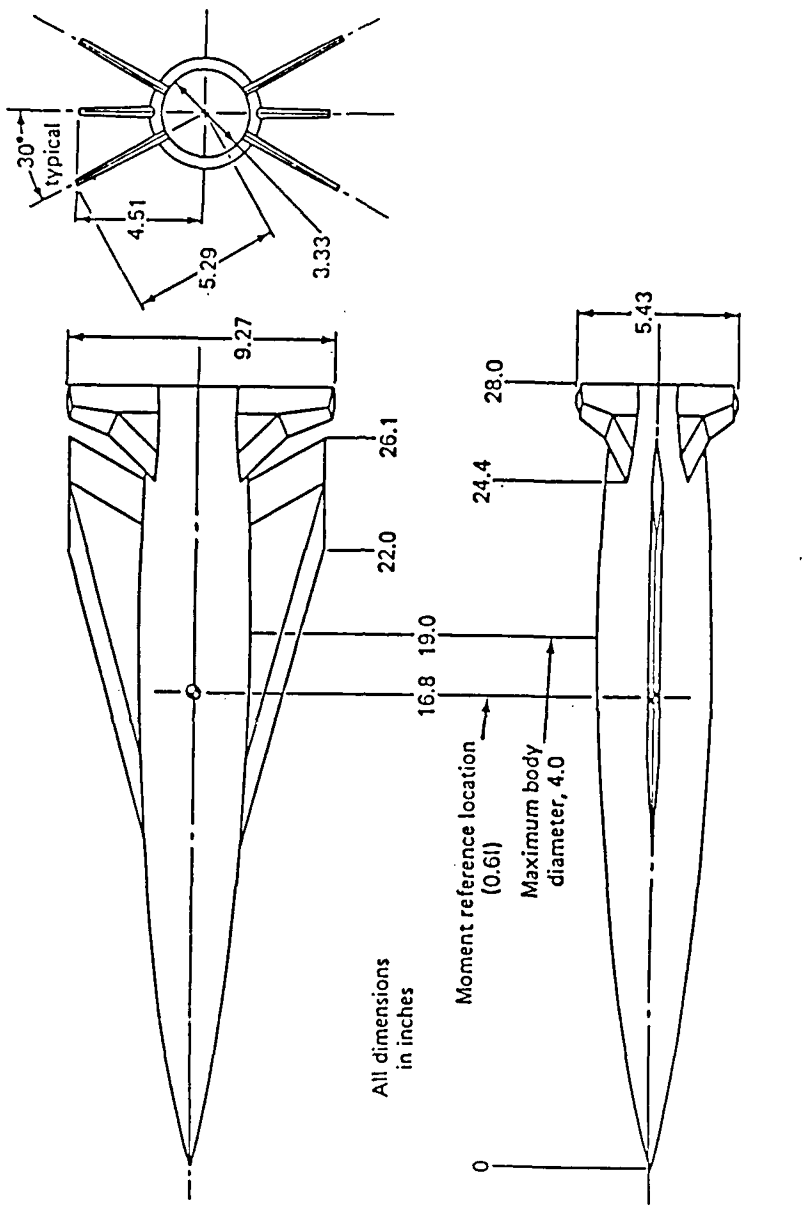

Figure A.1 Circular Configuration of the Missile

B. AUTOPILOT/AIRFRAME

Figure A.2 shows the block diagram of the autopilot used to develop the model of the pitch control channel of the missile.

The state equations that correspond to the autopilot are the following:

$$\mathring{x}_1 = -150.0\ x_1 + 150.0\ \eta_z$$

$$\mathring{x}_2 = 1.353\ x_1 + x_3 - 1.353\ \eta_{zc}$$

$$\mathring{x}_3 = -6.572\ x_1 - 5.0\ x_3 + 6.572\ \eta_{zc}$$

$$\mathring{x}_4 = -44.3316\ x_5 - 59.11\ x_6$$

$$\mathring{x}_5 = x_4 - 0.1482\ x_5 + 0.0395\ x_6$$

$$\mathring{x}_6 = -188.4\ x_6 + 188.4\ x_7$$

$$\mathring{x}_7 = -0.4608\ x_1 - 2.231\ x_2 - 0.3406\ x_3 + 2.231\ x_4$$
$$- 15.095\ x_5 - 20.13\ x_6 - 0.1430\ x_7 + 0.4608\ \eta_{zc}$$

The input for the autopilot is the commanded acceleration --- η_{zc}.

The values related to the aerodynamic are the following:

$C_{m\alpha} = -0.06$

$C_{m\delta p} = -0.08$

$C_{N\alpha} = 0.15$

$C_{N\delta p} = 0.04$

Dynamic Pressure	-	$\bar{q}$	----------	1650 lb/ft^2
Velocity	-	V	----------	3825.46 ft/sec

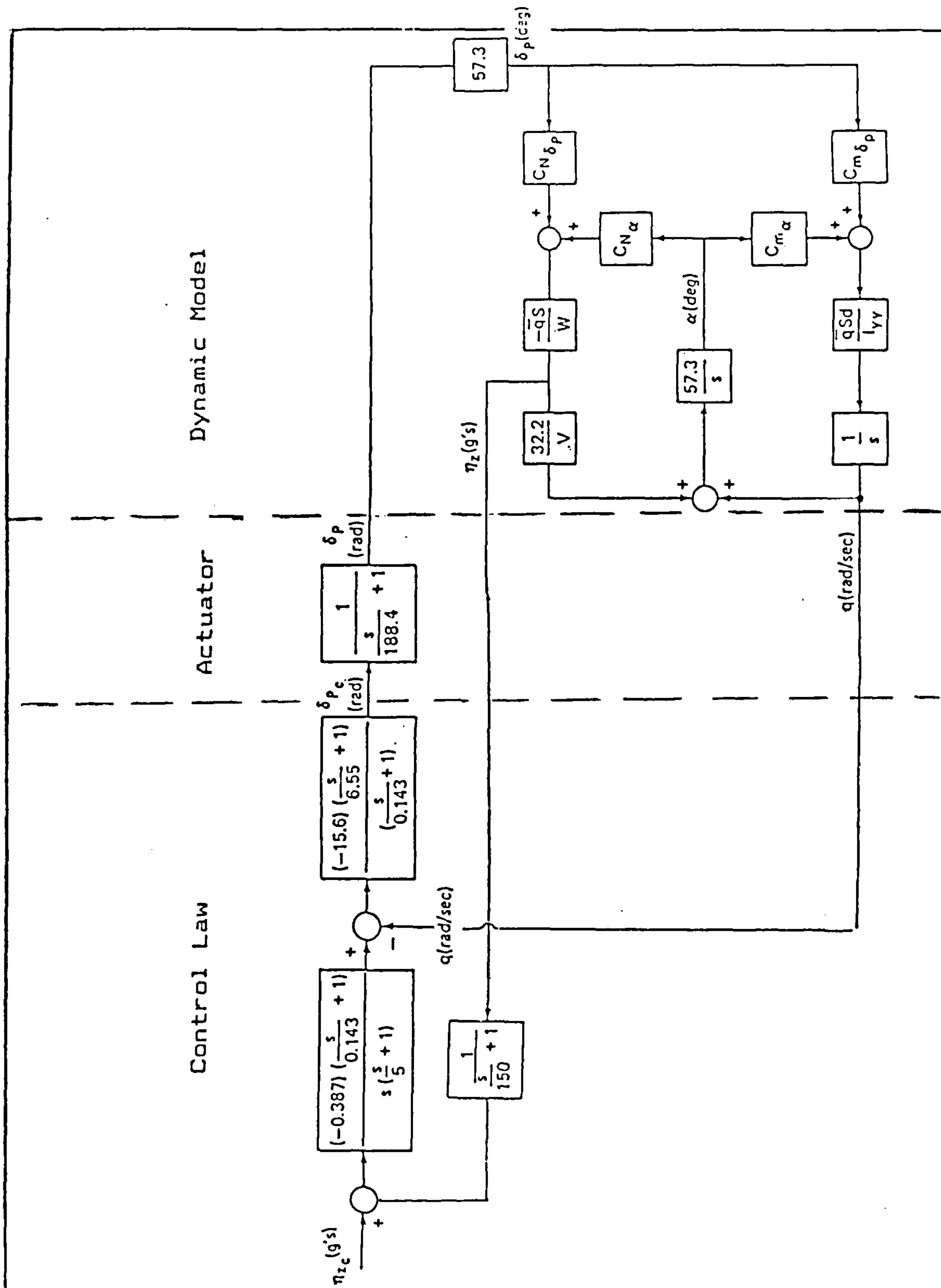

Figure A.2 Block Diagram of the Autopilot/Airframe

APPENDIX B

POPLAR PROGRAM

The computer output where the improvement in the input singular value was found is listed below.

```
FILE: LATSPROB OUT      A1                                                              PAGE 001

1

<<<<< RUN #  5001 >>>>>



 WMAX=   0.70000 INIT FREQ=   0.70000 FREQ STEP=   0.10000


 WEIGHT 1=   0.10000WEIGHT 2=   0.00000WEIGHT 3=   0.00000

 SVMINI=    0.60000 SVMINO=   0.50000  RJ=   0.10000  IDG=    0
 *** THE A PLANT MATRIX ***


-150.00000   0.00000   0.00000   0.00000**********-705.78003   0.00000   0.00000
   0.00000   0.00000
   1.35310   0.00000   1.00000   0.00000   0.00000   0.00000   0.00000   0.00000
   0.00000   0.00000
  -6.57200   0.00000  -5.00000   0.00000   0.00000   0.00000   0.00000   0.00000
   0.00000   0.00000
   0.00000   0.00000   0.00000   0.00000 -44.33162 -59.11000   0.00000   0.00000
   0.00000   0.00000
   0.00000   0.00000   0.00000   1.00000  -0.14820  -0.03950   0.00000   0.00000
   0.00000   0.00000
   0.00000   0.00000   0.00000   0.00000   0.00000-100.39999 100.39999   0.00000
   0.00000   0.00000
  -0.46080  -2.23000  -0.34050   2.25000 -15.09850 -20.13000  -0.14300   0.00000
   0.00000   0.00000
   0.00000   0.00000   0.00000   0.00000   0.00000   0.00000   0.00000 -50.00000
-495.00000   0.00000
   0.00000   0.00000   0.00000   0.00000   0.00000   0.00000   0.00000   0.00000
   0.00000   1.00000
   0.00000   0.00000   0.00000   0.00000 -17.44400  -4.70500   0.00000   0.00000
   0.00000   0.00000

 *** THE B CONTROL INPUT MATRIX ***


   0.00000   0.00000
  -1.35310   1.33440
   6.57200  -6.40100
   0.00000   0.00000
   0.00000   0.00000
   0.00000   0.00000
   0.46080  -0.45450
   0.00000 495.00000
   0.00000   0.00000
   0.00000   0.00000

 *** THE C OBSERVATION MATRIX ***


   1.00000   0.00000   0.00000   0.00000   0.00000   0.00000   0.00000   0.00000
   0.00000   0.00000
   0.00000   1.00000   0.00000   0.00000   0.00000   0.00000   0.00000   0.00000
   0.00000   0.00000
   0.00000   0.00000   1.00000   0.00000   0.00000   0.00000   0.00000   0.00000
   0.00000   0.00000
   0.00000   0.00000   0.00000   1.00000   0.00000   0.00000   0.00000   0.00000
   0.00000   0.00000
   0.00000   0.00000   0.00000   0.00000   1.00000   0.00000   0.00000   0.00000
```

```
  0.00000   0.00000
  0.00000   0.00000   0.00000   0.00000   0.00000   1.00000   0.00000   0.00000
  0.00000   0.00000
  0.00000   0.00000   0.00000   0.00000   0.00000   0.00000   1.00000   0.00000
  0.00000   0.00000
  0.00000   0.00000   0.00000   0.00000   0.00000   0.00000   0.00000   1.00000
  0.00000   0.00000
  0.00000   0.00000   0.00000   0.00000   0.00000   0.00000   0.00000   0.00000
  1.00000   0.00000
  0.00000   0.00000   0.00000   0.00000   0.00000   0.00000   0.00000   0.00000
  0.00000   1.00000

*** THE          F FEEDBACK MATRIX ***


  0.00000   0.00000   0.00000   0.00000   0.00000   0.00000   0.00000   0.04900
  0.98615   1.40440
  0.00000   0.00000   0.01000   0.00000   0.00000   0.00000   0.00000   0.00000
  0.00000   0.00000

*** THE ORDERED COMPLEX EIGENVALUES (INPUT)


 -159.00000  -19.89999
 -159.00000   19.89999
  -50.00000    0.00000
   -8.00000   -7.90000
   -8.00000    7.90000
   -7.00000    0.00000
   -5.00000    0.00000
   -1.00000    0.00000
   -1.00000    0.00000
   -1.00000    0.00000

*** THE INITIAL DATA IS ***


*** THE ORDERED COMPUTED EIGENVALUES


 -159.74232  -18.97508
 -159.74232   18.97508
  -50.02896    0.00000
   -8.63660   -7.89021
   -8.63668    7.89021
   -2.46024    0.00000
   -1.86206   -2.15078
   -1.86206    2.15078
   -0.50488    0.00000
   -0.14313    0.00000

*** THE ORDERED EIGENVECTORS


 -29.46394  -14.65100  -29.46394   14.65100    1.50392    0.00000   -4.43350   11.39655
  -4.43350  -11.39655    8.13620    0.00000    4.47925   -1.32631    4.47925    1.32631
   3.60236    0.00000    3.79569    0.00000
   0.26904    0.09535    0.26904   -0.09535    0.32167    0.00000   -1.06080   -1.36396
  -1.06080    1.36396    2.19106    0.00000    0.04739    0.00130    0.04739   -0.00130
   0.13223    0.00000    0.00672    0.00000
  -1.30795   -0.46341   -1.30795    0.46341   -1.56693    0.00000    5.29377    6.64100
   5.29377   -6.64100  -11.29992    0.00000   -4.42302   -4.25205   -4.42302    4.25205
  -0.90271    0.00000  -81.51242    0.00000
  -0.04065   -0.36439   -0.04065    0.36439   -0.20462    0.00000   -7.73727   -1.42416
  -7.73727    1.42416    1.37471    0.00000    0.72477    0.53061    0.72477   -0.53061
   0.09012    0.00000   -0.00145    0.00000
   0.00054    0.00197    0.00054   -0.00197    0.00551    0.00000    0.56700   -0.36376
```

```
    0.56700     0.36376    -0.58603     0.00000    -0.31322     0.00299    -0.31322    -0.00299
   -0.25464     0.00000    -0.26868     0.00000
   -0.01491    -1.00186    -0.01491     1.00186    -0.24503     0.00000    -1.34566    -0.96810
   -1.34566     0.96810     0.49679     0.00000     0.23044    -0.01917     0.23044     0.01917
    0.19194     0.00000     0.20211     0.00000
   -0.10317    -0.15088    -0.10317     0.15088    -0.17996     0.00000    -1.34360    -0.86653
   -1.34360     0.86653     0.49030     0.00000     0.23506    -0.02171     0.23506     0.02171
    0.19143     0.00000     0.20199     0.00000
   -0.06046    -0.00966    -0.06046     0.00966  -251.15575     0.00000    -1.28696    -1.43904
   -1.28696     1.43904   -12.59149     0.00000     2.72214     5.84836     2.72214    -5.84836
 -140.76167     0.00000 -1828.26782     0.00000
    0.00004     0.00018     0.00004    -0.00018     0.00042     0.00000     0.07750     0.03327
    0.07750    -0.03327     1.32212     0.00000    -0.24593    -0.51431    -0.24593     0.51431
   14.08348     0.00000   184.95924     0.00000
   -0.00381    -0.02884    -0.00381     0.02884    -0.02110     0.00000    -0.40679    -0.89884
   -0.40679     0.89884    -3.25274     0.00000    -0.64822     1.48662    -0.64822    -1.48662
   -7.11060     0.00000   -26.47536     0.00000
```

```
*** THE       F FEEDBACK MATRIX ***


  0.00000   0.00000   0.00000   0.00000   0.00000   0.00000   0.00000   0.04908
  0.98615   1.40440
  0.00000   0.00000   0.01000   0.00000   0.00000   0.00000   0.00000   0.00000
  0.00000   0.00000


FREQUENCY     MIN ADD IN SV    2ND ADD IN SV   MAX ADD IN SV  MIN PROD ADD IN SV
  0.70000        0.55284          3.25329         3.25329         1.34110

   FREQ      SVADMO    SVADXO    SVMIN     SVMIX     SVMMO     SVMXO

  0.70000   0.05114  35.16896   0.522202221.19727   2.00000   2.00000
1
```

```
   AAAAA    DDDDDD     SSSSSS
  A     A   D     D   S
  A     A   D     D   S
  AAAAAAA   D     D    SSSSS
  A     A   D     D         S
  A     A   D     D         S
  A     A   DDDDDD     SSSSSS
```

F O R T R A N   P R O G R A M

F O R

A U T O M A T E D   D E S I G N   S Y N T H E S I S

V E R S I O N   1.00

CONTROL PARAMETERS

ISTRAT = 5 IOPT = 3 IONED = 3 IPRINT = 1000
IGRAD = 0 NDV = 7 NCON = 0
1

OPTIMIZATION RESULTS

OBJECTIVE FUNCTION VALUE 0.18698E+00

DESIGN VARIABLES

VARIABLE	LOWER BOUND	VALUE	UPPER BOUND
1	-0.10000E+05	0.59317E-01	0.10000E+05
2	-0.10000E+05	-0.11494E+01	0.10000E+05
3	-0.10000E+05	0.30785E-01	0.10000E+05
4	-0.10000E+05	-0.16953E+01	0.10000E+05
5	-0.10000E+05	-0.90698E+01	0.10000E+05
6	-0.10000E+05	0.31414E+00	0.10000E+05
7	-0.10000E+05	0.70745E+01	0.10000E+05

FUNCTION EVALUATIONS = 235

*** THE OPTIMIZER COMPUTED OUTPUT IS ***

*** THE ORDERED COMPUTED EIGENVALUES

-159.73711	-19.04807
-159.73711	19.04807
-56.03259	0.00000
-8.04688	-7.97487
-8.04688	7.97487
-7.06465	0.00000
-5.00031	0.00000
-0.37434	-0.39750
-0.37434	0.39750
-0.14305	0.00000

*** THE ORDERED EIGENVECTORS

-27.97853	-12.15030	-27.97853	12.15030	1.71545	0.00000	-0.61352	-0.96862
-0.61352	0.96862	-2.93255	0.00000	-2.22260	0.00000	0.46230	-0.57794
0.46230	0.57794	-3.69572	0.00000				
0.27374	0.08283	0.27374	-0.08283	0.34835	0.00000	0.18800	-0.03511
0.18800	0.03511	14.59894	0.00000	-2.09061	0.00000	0.03296	0.00429
0.03296	-0.00429	-0.00719	0.00000				
-1.33078	-0.40259	-1.33078	0.40259	-1.69691	0.00000	-0.91973	0.18036
-0.91973	-0.18036	-72.37257	0.00000	14.44669	0.00000	-0.18979	0.01996
-0.18979	-0.01996	81.40660	0.00000				
-0.06026	-0.34808	-0.06026	0.34808	-0.30821	0.00000	0.50392	-0.49214
0.50392	0.49214	-1.91382	0.00000	-0.08340	0.00000	0.02459	0.00252
0.02459	-0.00252	-0.00666	0.00000				
0.00060	0.00187	0.00060	-0.00187	0.00597	0.00000	-0.00022	0.06230
-0.00022	-0.06230	0.27422	0.00000	0.17743	0.00000	-0.03261	0.04087
-0.03261	-0.04087	0.26157	0.00000				
-0.04631	-0.96238	-0.04631	0.96238	-0.26536	0.00000	0.13517	-0.04574
0.13517	0.04574	-0.43442	0.00000	-0.20908	0.00000	0.02460	-0.03047
0.02460	0.03047	-0.19588	0.00000				
-0.10053	-0.14151	-0.10053	0.14151	-0.19489	0.00000	0.12746	-0.04950
0.12746	0.04950	-0.41811	0.00000	-0.20337	0.00000	0.02448	-0.03046
0.02448	0.03046	-0.19600	0.00000				
-0.06095	-0.00637	-0.06095	0.00637	-243.04360	0.00000	0.19754	-0.04027
0.19754	0.04027	0.99126	0.00000	-0.67280	0.00000	20.11691	14.34287
20.11691	-14.34287	1782.69556	0.00000				

```
    0.00005    0.00017    0.00005   -0.00017    0.00046    0.00000   -0.00693    0.00486
   -0.00693   -0.00486   -0.05590    0.00000   -0.08320    0.00000   -2.02647   -1.42194
   -2.02647    1.42194 -180.37170    0.00000
   -0.00472   -0.02755   -0.00472    0.02755   -0.02285    0.00000    0.09455    0.01615
    0.09455   -0.01615    0.39554    0.00000    0.42267    0.00000    0.19326    1.33799
    0.19326   -1.33799   25.00348    0.00000
```

*** THE   P FEEDBACK MATRIX ***

```
  0.03933  -1.14944   0.03070  -1.69532  -9.04974   0.31539   7.07442   0.04900
  0.90615   1.40440
  0.00000   0.00000   0.01000   0.00000   0.00000   0.00000   0.00000   0.00000
  0.00000   0.00000
```

```
FREQUENCY    MIN ADD IN SV   2ND ADD IN SV   MAX ADD IN SV  MIN PROD ADD IN SV
  0.70000       0.67511         2.06293        2.06293        1.39025

   FREQ      SVADMO    SVADXO     SVMIN     SVMIX     SVMMO     SVMXO

  0.70000   0.00790 242.09914   0.710491751.05767   2.00000   2.00000
```

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